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Research Article

Ordinary-level learners' errors in solving simultaneous linear equations at a Zimbabwean secondary school

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This study sought to reveal the ordinary level learners' errors in solving simultaneous linear equations using any method of their choice at a secondary school in Chikomba district of Zimbabwe. The study used convenience sampling to select 100 learners. In the qualitative methodology adopted, data was gathered using transcripts of an achievement test and structured interviews. The study used content analysis of selected tasks to expose learners' mental construction when solving the simultaneous linear equations. The Ordinary Level learners consistently made procedural errors when they used the substitution, the elimination, and the matrix methods. The researchers recommend future studies to build on the loopholes of these ever-present methods and invest in hybrid ways of solving simultaneous linear equations.

Keywords: Errors, Misconceptions, Simultaneous linear equations

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1. Introduction

For the five years, 2017-2021 schools in Chikomba District witnessed a decline in pass rates. The relatively low pass rates in Mathematics could also be attributed partly to the introduction of teacher-based assessment modes, and the errors and misconceptions committed by learners. Considering that Continuous Assessment Learning Activities, CALAs, (Ministry of Primary and Secondary Education, 2023) contribute a weight of 30% to the final mark, errors and misconceptions in any topic may contribute substantially to the final grade in national examinations.

Research has been carried out to explore errors and misconceptions in Math topics such as Algebra (Mulungye, 2016; Welder, 2012); fractions (Alghazo & Alghazo, 2017; Low et al., 2020); differentiation (Chikwanha et al., 2022); linear ordinary differential equations (Msomi & Bansilal, 2022); and derivatives of trigonometric (Siyepu, 2015). Others have studied learners repeating Grade 11 (Roselizawati et al., 2014); and the errors and misconceptions in solving Simultaneous Equations (Johari & Shahrill, 2020; Ugboduma, 2012). These studies have not been exhaustive in the kinds and nature of errors.

Research has shown that those students, taught by teachers with both knowledges of content and of misconceptions, stand to benefit more than those who are taught by teachers with content knowledge only (Sadler et al., 2013). We believe it is essential that teachers must be aware of these common errors and misconceptions so that they are able to challenge and help learners to reconstruct conceptions. The teachers can use their knowledge of errors and misconceptions to make informed instructional decisions (Banerjee &

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Subramaniam, 2012; Welder, 2012). It is essential that teachers are conscious of the potential errors and misconceptions learners hold regarding methods of solving simultaneous equations.

Simultaneous equations are two linear equations with two unknown variables that have the same solution. The three methods mainly used to solve simultaneous equations are; the addition or subtraction method (also known as the elimination method), the substitution method and the graphical solution. Furthermore, matrices can be used to solve simultaneous equations but is not a popular method with both learners and teachers. The learners who opt to use the matrix method would require an understanding of matrix multiplication, determinant, inverse, and identity. The topic 'Simultaneous Equations' is often perceived as complex, demanding, and requires a plethora of algebraic processing skills to find the correct solution (Ugboduma, 2012). The heterogeneous nature of simultaneous equations complicates matters and could be why, most learners perform poorly on such questions in the examinations.

This paper sought to answer the following research questions:

RQ 1) What are the errors displayed by learners when solving simultaneous linear equations?

RQ 2) What is the prerequisite knowledge required by learners to successfully solve simultaneous linear equations?

2. Literature Review

Students make errors and display misconceptions when learning simultaneous equations (Msomi & Bansilal, 2022; Ugboduma, 2012). From the encountered definitions, errors and misconceptions are related but, in a way, different. They seem to be interwoven, but an error is a slip, mistake, or deviation from accuracy (Makonye, 2014), and can be systematic or unsystematic (Luneta & Makonye, 2010). The unsystematic errors are defined as intended, non-recurring wrong answers which learners can readily correct by themselves, whereas systematic errors are recurrent incorrect responses methodically constructed and produced across space and time (Luneta & Makonye, 2010). Errors can also be intentional or unintentional (Msomi & Bansilal, 2022). Lucariello and Naff (2014) explains:

> erroneous understandings are termed alternative conceptions or misconceptions (or intuitive theories) … alternative conceptions (misconceptions) are not unusual. In fact, they are a normal part of the learning process… we quite naturally form ideas from our everyday experience, but obviously not all the ideas we develop are correct (cited in Ellis, 2021, p.4).

Misconceptions are conceptual structures, which make sense in relation to one's current knowledge but are not aligned with the conventional mathematical wisdom (Luneta & Makonye, 2010; Siyepu, 2013). A misconception is a misapplication of a rule, an over or under generalization or alternative conception of the situation (Hansen, 2006). As a matter of fact, misconceptions are termed 'beautiful windows' which exist in the student's mind allowing for deeper learning when intentionally elicited, explored, revised, and resolved through students' sense-making (Ellis, 2021).

Furthermore, misconceptions are often implanted in errors displayed. Literature accessed shows that there are connections and likenesses in the different ways of grouping errors (Siyepu, 2015; Orton, 1983). The distinction between an error and a misconception is blurred. For example, misapplication of a rule can be viewed as a misconception (Hansen, 2006), yet others classify it as an applicability error (Clark, 2012). Carelessness or silly mistakes are examples of technical errors (Godden et al., 2013; Tendere & Mutambara, 2020). Misconceptions are revealed in errors evident in problem solving (Mutambara & Bansilal, 2019; Siyepu, 2013). The current study focused on the errors when solving simultaneous linear equations.

Chikwanha et al. (2022) explored Advanced Level learners' errors and misconceptions in differentiation. They adopted a qualitative case study strategy; qualitative data used was generated from test scripts, syllabus documents, questionnaires with open-ended items, and focus group discussions. Chikwanha et al. (2022) employed content analysis to disclose the errors and misconceptions made by students as they solve problems in differentiation. Our current study also used content analysis to establish the prerequisite knowledge required to solve simultaneous linear equations.

Johari and Shahrill (2020) investigated the common errors in learning simultaneous equations displayed by Year 9 students in one of the elite government schools in Brunei Darussalam. Their study revealed four main factors that were impeding learning mathematics; complicating subject, wrong substitution of the subject, mathematical error and irrational error in solving the question. The factors were derived from recurring patterns and occurrences in solving simultaneous equations. Johari and Shahrill (2020) revealed that the vigorous and heterogeneous nature of simultaneous equations was intimidating learners. Our research gap was the lack of reports on errors and misconceptions from numerous methods used to solve simultaneous equations like graphical and matrix methods, which are in the Zimbabwe O-Level Mathematics syllabus 2015.

3. Research Methodology

The research used a qualitative study because of the need to present detailed descriptions of learners' errors and misconceptions in solving simultaneous equations. Adopting a qualitative research design helped the researchers in that it allowed the gathering of in-depth information on the errors and misconceptions as they surfaced in the learners' attempts to solve simultaneous linear equation. Several researchers have successfully used qualitative research methodology (Chikwanha et al., 2022; Johari & Shahrill, 2020; Msomi & Bansilal, 2022; Siyepu, 2015).

The study population was 100 learners from two ordinary level classes at a secondary school in Chikomba district, Zimbabwe. The ordinary level learners were Form IV (FIV) learners, equivalent to Year 11 students or GCSE candidates. Convenient sampling was used to select one school in Chikomba district. All 100 ordinary level (FIV) learners at the school participated in the study.

Data was collected by administering a test developed using ZIMSEC O-Level past examination papers. We considered these items suitable because they met ZIMSEC standards in terms of content validity and reliability in discriminating learners of different abilities. Furthermore, we used a grid specification method to ensure that the first four levels of Bloom's taxonomy namely remembering, understanding, applying, and analyzing (Johari & Shahrill, 2020) were included. A twenty-minute assessment test comprising four items (refer to Table 1) on simultaneous linear equations was administered, and the transcripts were used to find out the errors made. Ten students were purposively selected for interviews to determine the misconceptions held when they solved simultaneous linear equations. The interviews were transcribed and analyzed to reveal the prerequisite knowledge required to successfully solve the simultaneous linear equations.

Table 1
Test items

Item number	*Questions*
1	$x + 0.5y - 1 = 0$ $3x - 4y = 14$
2	$3x - y = 2$ $5x - 2y = 0$
3	$x + y = 5\frac{1}{2}$ $x - 2y = 2\frac{1}{2}$
4	$2x + 3y = 11$ $3x + 12 = 5$

Note. Adopted from ©ZIMSEC PAST EXAMINATIONS PAPERS

4. Results and Discussion

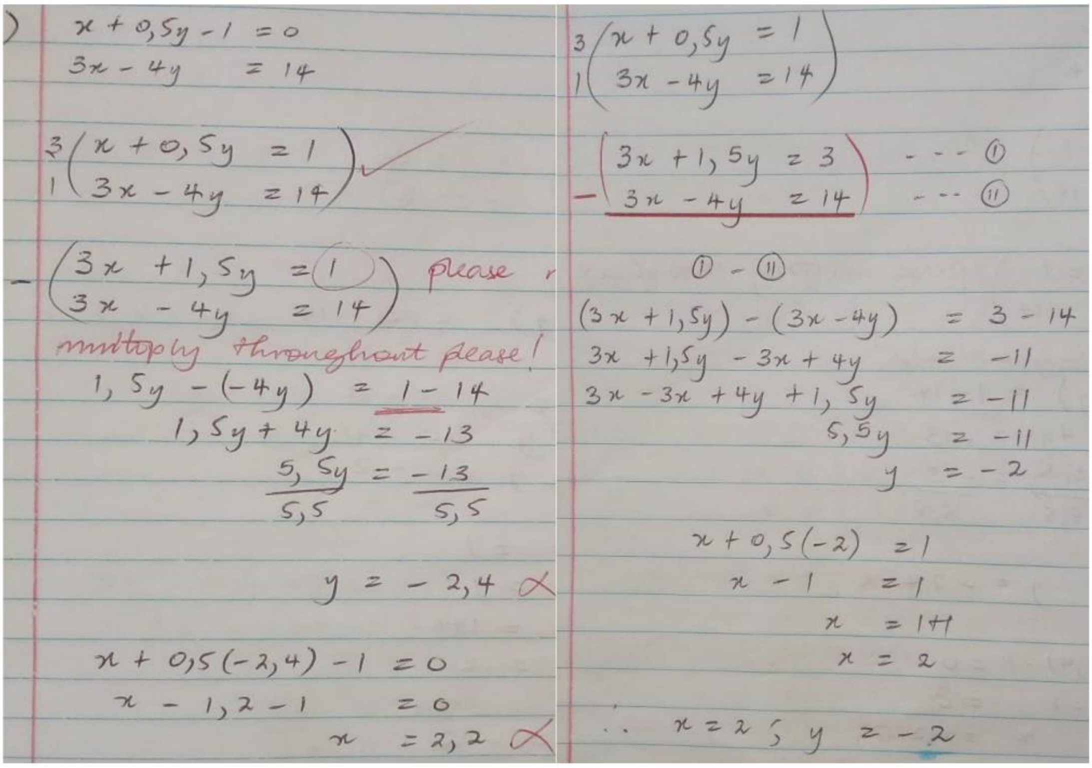
The researchers looked for recurring patterns and occurrences of errors and misconceptions. Data analysis revealed eight categories of errors; Irrational Error (IE), simple Mathematical Error (ME), Inverse and Determinant error (ID), Complicating the Subject (CS), Wrong Substitution (WS), Wrong Choice of signs (WC), Sequential Error (SE), and Wrong Multiplication of matrices (WM). Some of these errors are catalysts for subsequent errors.

4.1. Errors and Misconceptions

4.1.1. Irrational Error (IE), e.g., failure to expand

The spread below, Figure 1, shows learners' errors and misconceptions on the left-hand side. On the right-hand side is shown the correct procedure to arrive at the solution using the elimination method. The learner did not multiply 3 with both LHS and RHS of the first equation. Instead of $3(x + 0.5y = 1)$ to give $3x + 1.5y = 3$ the learner wrote $3x + 1.5y = 1$. Beyond this step the learner carried out the procedure correctly, hence carried forward error.

Figure 1
Sample of learner's work showing an error in expansion



When learners use the method of elimination, they usually make correct choices for multiplication to make specific coefficients from the two equations equal. An error often made by learners is the failure to multiply all the terms from the equations as indicated in Figure 1. In Figure 1, the learner successfully chose the number 3 to multiply equation $x + 0.5y - 1 = 0$ and correctly rearranged to $x + 0.5y = 1$. The learner chose to swap and multiply coefficients from both equations. However, on multiplying $3(x + 0{,}5y = 1)$ the learners did not multiply 3 x 1 on the right-hand side of the equation. We created the category 'Irrational Error, (IE), that is, failure to expand. Johari and Shahrill (2020) classified similar errors as simple mathematical errors of expansion due to carelessness.

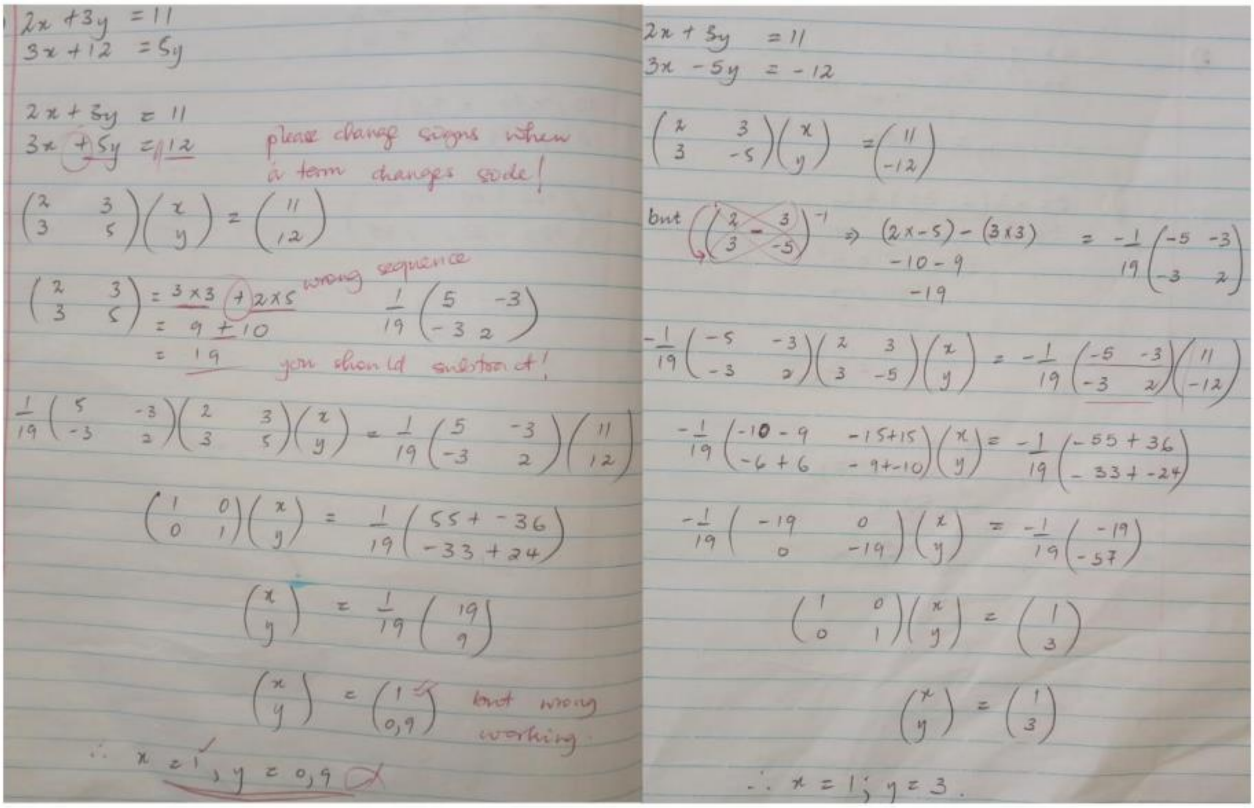
4.1.2. Simple Mathematical Errors (ME) in rearranging terms of an equation when using matrices

The learners in our study displayed errors in the initial phase of trying to solve SLEs e.g., the mistake in rearranging terms of an equation. Such errors later led to difficulties in using matrices to solve simultaneous linear equations (see Figure 2). Looking at the nature of Test Item 4; $(2x + 3y = 11;\ 3x + 12 = 5y)$, consisting of simple linear equations, learners can apply any of the three methods. However, on this other equation $(3x + 12 = 5y)$ when a learner chose the matrix method it has to be arranged and match the unknowns to that of the other equation. An error was resulting from a failure to rearrange terms was classified as mathematical error (Johari & Shahrill, 2020).

The learner's failure to rearrange terms manifested in the later stages of solving the SLEs. One learner failed to change signs when swapping the terms from the right-hand side (RHS) of the equal sign to the left-hand side (LHS) and vice versa. Whenever the terms swap sides, their signs are supposed to change as well such that it becomes $(3x - 5y = -12)$. This error compromised the initial equation. We categorized error in rearranging terms as simple Mathematical Error (ME).

Simple mathematical errors, as in Figure 2, are common as reported in literature (Johari & Shahrill, 2020). Such errors include mistakes of expansion, rearranging, algebraic manipulations, and wrong multiplication of directed numbers. Learners make mistakes when carrying mathematical operations like addition, subtraction, multiplication and division.

Figure 2
Sample of learner's work on item 4 showing error in rearranging

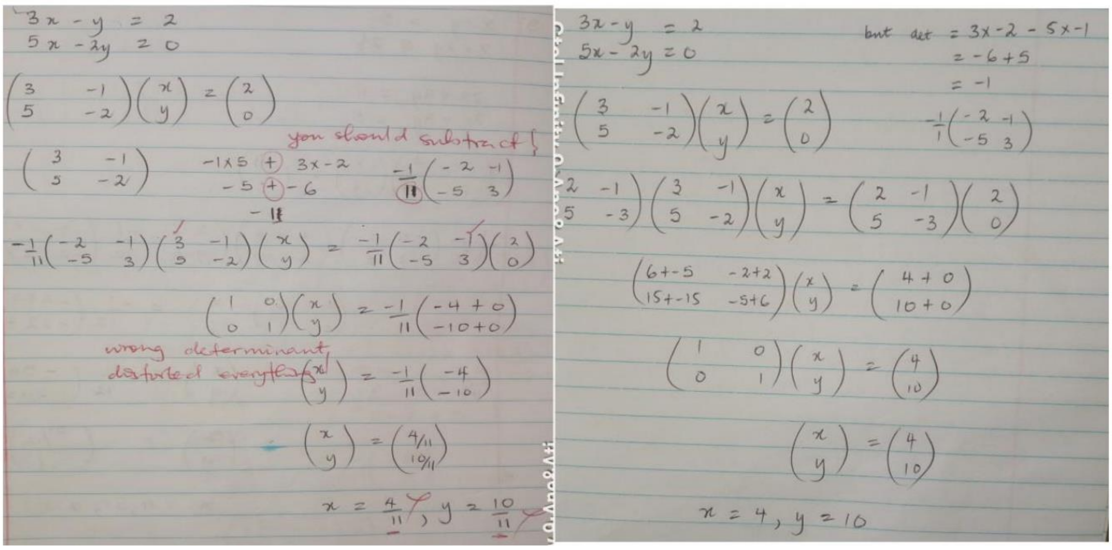


4.1.3. Failure to find inverse and determinant (ID) when solving simultaneous linear equations using matrices

We found out that learners display errors when solving simultaneous equations using the matrices method, as shown in Figure 3 below. The determinant of 2 x 2 matrix is obtained by subtracting the product of elements on the non-principal diagonal from the product of the elements of the principal diagonal. The learner erred, as shown in Figure 3, by not subtracting as required and carried forward the error (of wrong determinant).

The conversation with learners revealed the mix-up (error) of major and minor diagonal. Therefore, knowledge and understanding of matrices is a prerequisite to use the matrix method in solving simultaneous equations.

Figure 3
Learner's error on finding the determinant

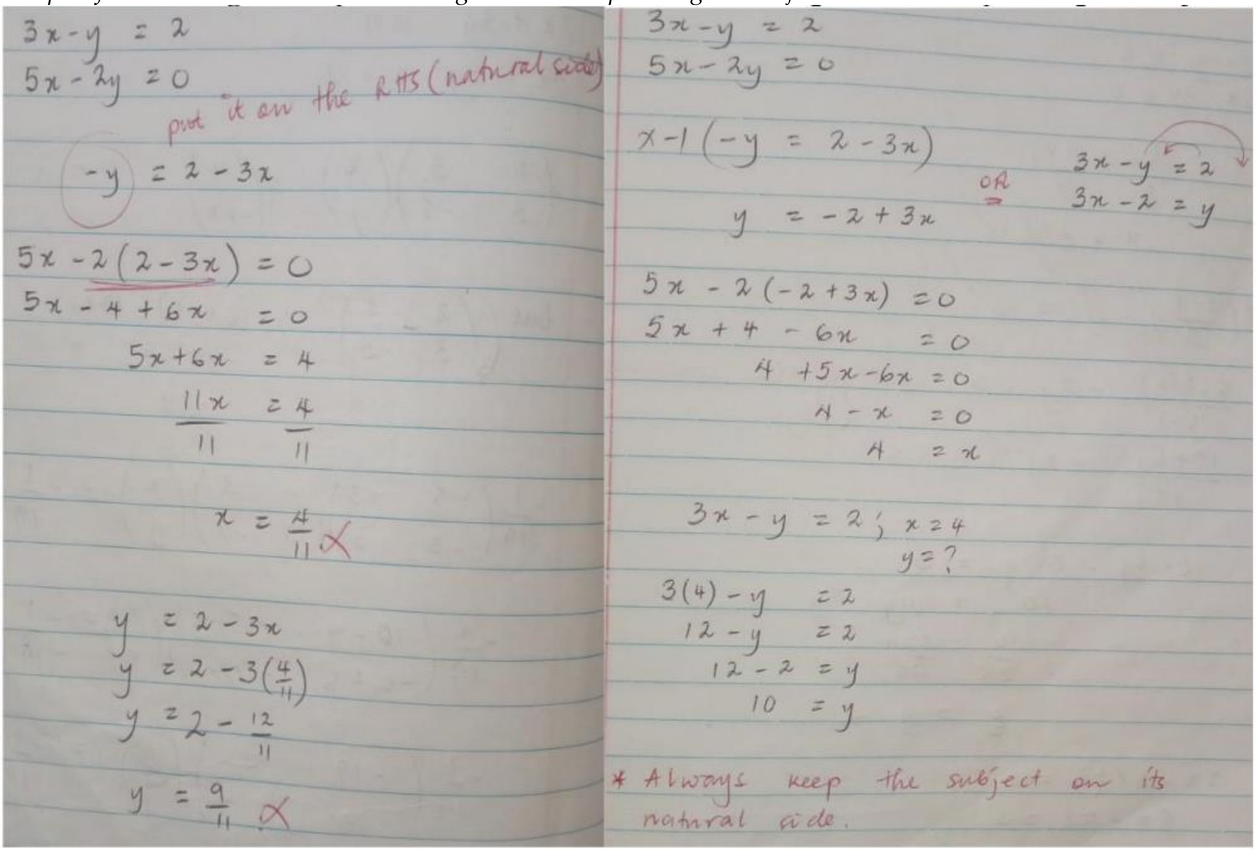


Our category of **I**nverse and **D**eterminant (ID) refers to failure to find the determinant and writing the inverse of the coefficient matrix when solving simultaneous linear equations. The ID errors manifest as learners attempt to find the determinant and fail to write the inverse of the coefficient matrix. When learners reintroduce the inverse on both sides, they often get wrong answers. For example, on item 4, $(2x + 3y = 11)$ and $3x - 5y = -12$, the coefficient matrix becomes $\begin{bmatrix} 2 & 3 \\ 3 & -5 \end{bmatrix}$, most learners who attempted this made a subsequent error on determinant, that is, $(\ 2 \times -5) + (3 \times 3)$. The correct way for the determinant is $(2 \times -5) - (3 \times 3)$, where one is supposed to subtract the product of the diagonals (minor from major) and the correct inverse becomes $\frac{-1}{19}\begin{bmatrix} -5 & -3 \\ -3 & 2 \end{bmatrix}$. There is also evidence in the literature that learners show difficulties in finding the determinant (e.g., Chagwiza et al., 2021).

4.1. 4. Difficulties of complicating the subject (CS) when solving simultaneous equations using the elimination method

We categorized errors in Figure 4 below as Complicating Subject (CS). For example, in Item 2, instead of using the simple subject $3x - 2 = y$, they used $-y = 2 - 3x$. Some made x the subject and used $x = \frac{2+y}{3}$. Thus, complicating the subject, which causes subsequent errors later in the working. Our study found out that learners complicated the subject by leaving y on its unnatural side, where the subject is negative (Channon et al., 2004). The expression $(-y = 2 - 3x)$ is not wrong but demands a particular skill when substituting it into the other equation. Such unnecessary complications usually increase the risk of participants being careless as shown in Figure 4, where the learner did not substitute correctly. Typically, a lot of errors are usually made when the subject is made into a complicated fraction form (Johari & Shahrill, 2020; Low et al., 2020).

Figure 4
Sample of learner's work on item 2 showing error in complicating the subject



Interviews revealed that learners encountered difficulties when working with fractions. For example, in answering Test Item 2, the learner complicated the subject and ended up with fractions. The simultaneous equations did not require the use of fractions, implying conceptual misunderstanding on the part of the learner.

4.1.5. Wrong substitution when using the elimination method

Several students were able to change the subject but wrongly substituted it into the other equation. They failed to follow the format of the other equation religiously. In the case of the item 3 below, there are a handful of coefficients from both equations which are 1.

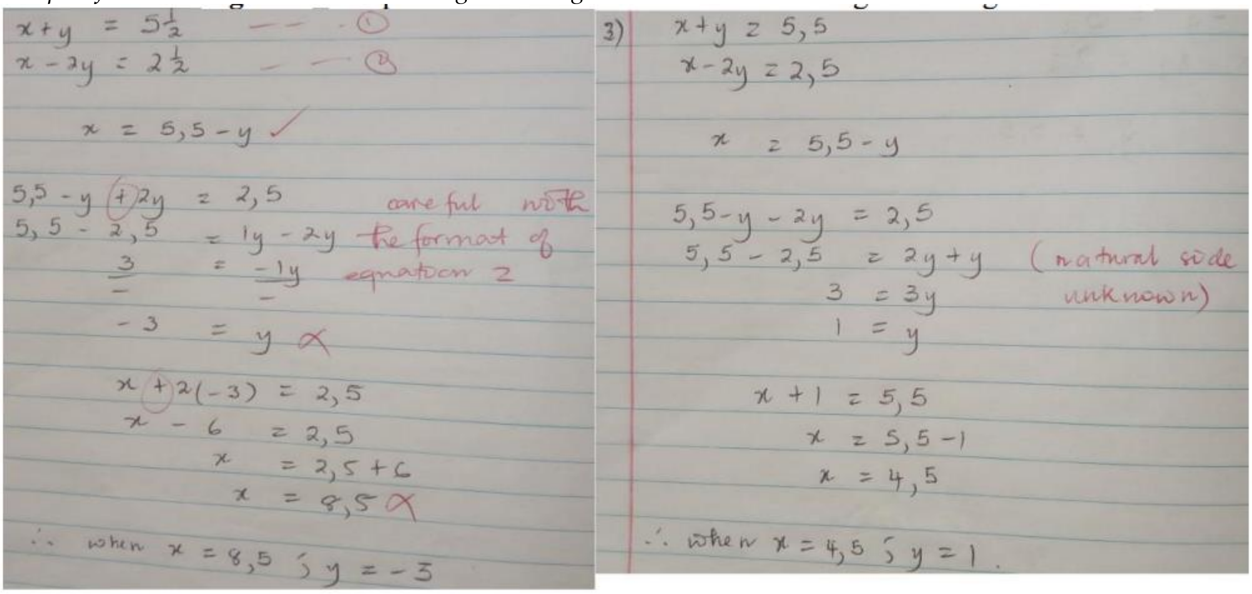
$$x + y = 5\frac{1}{2}$$
$$x - 2y = 2\frac{1}{2}$$

Some students chose to use the substitution method and proceeded to change the subject from one of the equations well. However, the same students were careless in looking for the format of the other equation and did not substitute correctly.

Test item 3 could be solved using the elimination method, that is addition/subtraction method. The response in Figure 5 below suggest that learners encountered difficulties when solving the simultaneous linear equation;

In Figure 5, the learner was able to isolate **x**. However, the same learner failed to substitute **x** correctly in the second equation. The learner treated the expression negative **2y** as positive. The rest of the solution carried forward the error. The response, in Figure 5, is an example of wrong substitution.

Figure 5
Sample of learner's work on item 3 showing the wrong substitution



We categorized difficulties displayed in Figure 5 as Wrong Substitution (WS). The learners failed to follow the format of the other equation in deletion of coefficients. As shown in Figure 5 above, when substituting $x = 5.5$ - y into $x - 2y = 2.5$, the learners who substituted it as $5.5 - y + 2y = 2.5$ failed to follow the format of the other equation carefully. Our finding is consistent with Johari and Shahrill (2020), who described wrong substitution as literally substituting subject wrongly or only substituting the subject partially.

4.1.6. The error of wrong choice (WC) of sign when using the elimination method to solve simple linear equations

Test item 4 could also be solved using the elimination method. The simultaneous equations:

$$2x + 3y = 11$$
$$3x + 12 = 5y$$

Using direct addition or subtraction of one equation from another will not work. Such simultaneous linear equations require equivalent equations. This could be achieved by rearranging second equation followed by simultaneously multiplying first equation by 3 and the second equation by 2 to give;

$$2x + 3y = 11$$
$$3x - 5y = -12$$

$$6x + 9y = 33$$
$$6x - 10y = -24$$

As shown in Figure 6, the learner did this right. However, the learner fails to subtract the second equation from the first.

$$9y - (-10y) = 33 - (-24)$$
$$19y = 57$$

Beyond this step. the learner carried forward the error of $y = -9$.

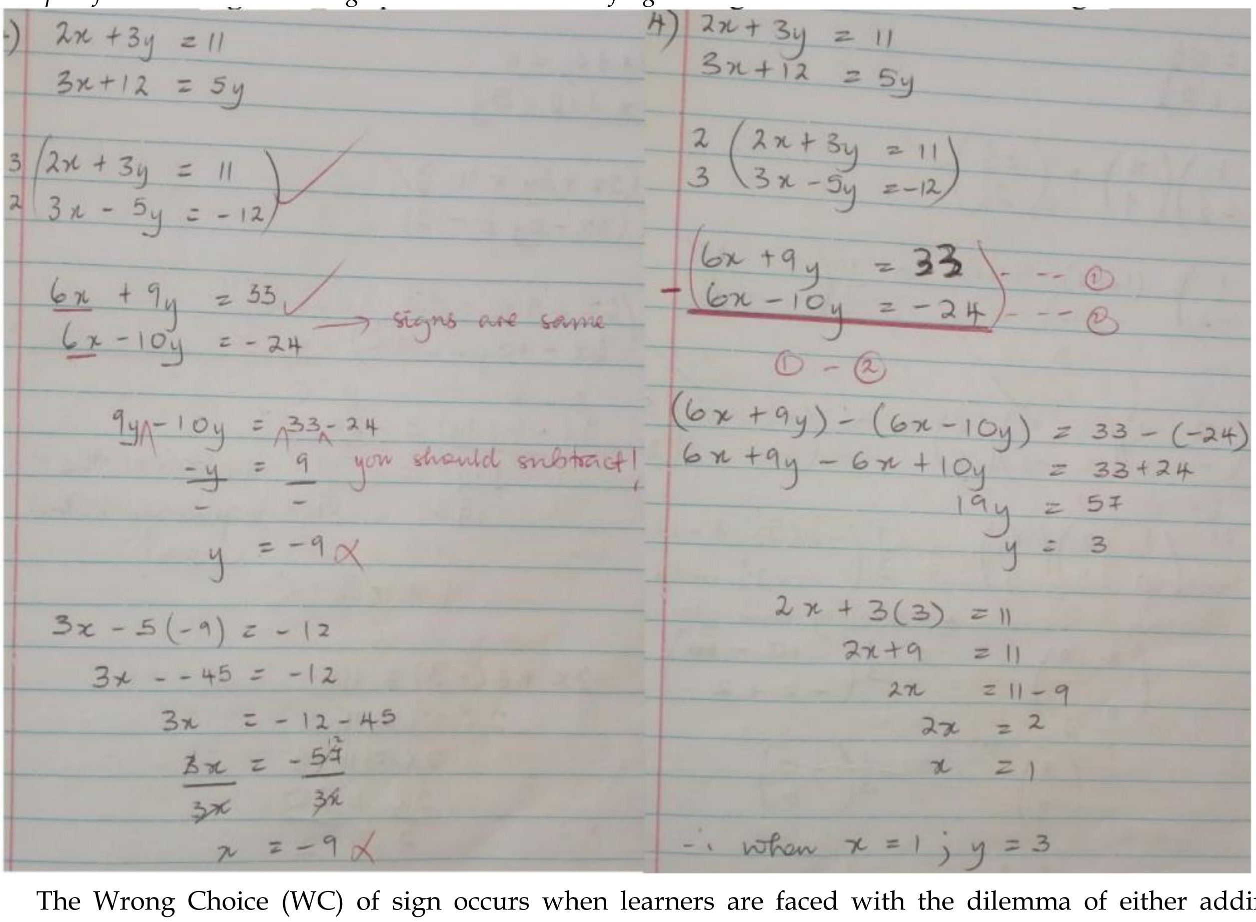
The learner's choice of either adding the two equations or subtracting one equation from the other rests on the signs of the focal unknown. In our study, more than half of the learners who chose the elimination method displayed errors when choosing the correct sign for eliminating the chosen unknown. Others were wrong in selecting the focal unknown, and avoided the complicated nature of signs of the focal unknown.

The nature of item 4 ($2x + 3y11;\ 3x + 12 = 5y$) was suitable for the elimination method, too. The second equation can be rearranged into ($3x - 5y = -12$) for easier manipulation. This poses a choice now on which unknown to eliminate. Some learners would multiply to get $6x + 9y = 33$ and $6x - 10y = -24$. In this case, the coefficient of x is positive 6 in both equations. To eliminate x the learners were supposed to subtract equations paying attention to sequence.

Another option is to eliminate y by multiplying the lowest common multiple of the coefficients of y on both equations for instance $5(2x + 3y = 11)$; $3(3x - 5y = -12)$ to get $10x + 15y\ 33$ and $9x - 15y = -36$. In this instance, y is our focal unknown. The coefficients of y are +15 and -15, differing on sign only, hence the need to add the two equations. We interpreted the response in Figure 6 as an example of a procedural error (Orton, 1983).

Figure 6

Sample of learner's work showing an error in the choice of sign



The Wrong Choice (WC) of sign occurs when learners are faced with the dilemma of either adding (instead of subtracting) or vice versa for elimination purposes. The signs of the coefficients which they want to eliminate determine the choice of the sign for elimination for example in item ($x + y = 5\frac{1}{2}$; $x - 2y = 2\frac{2}{3}$) both the coefficients of x are positive and hence the need to subtract the two equations and eliminate x. Failure to make the correct choice and respect it may lead to wrong solution.

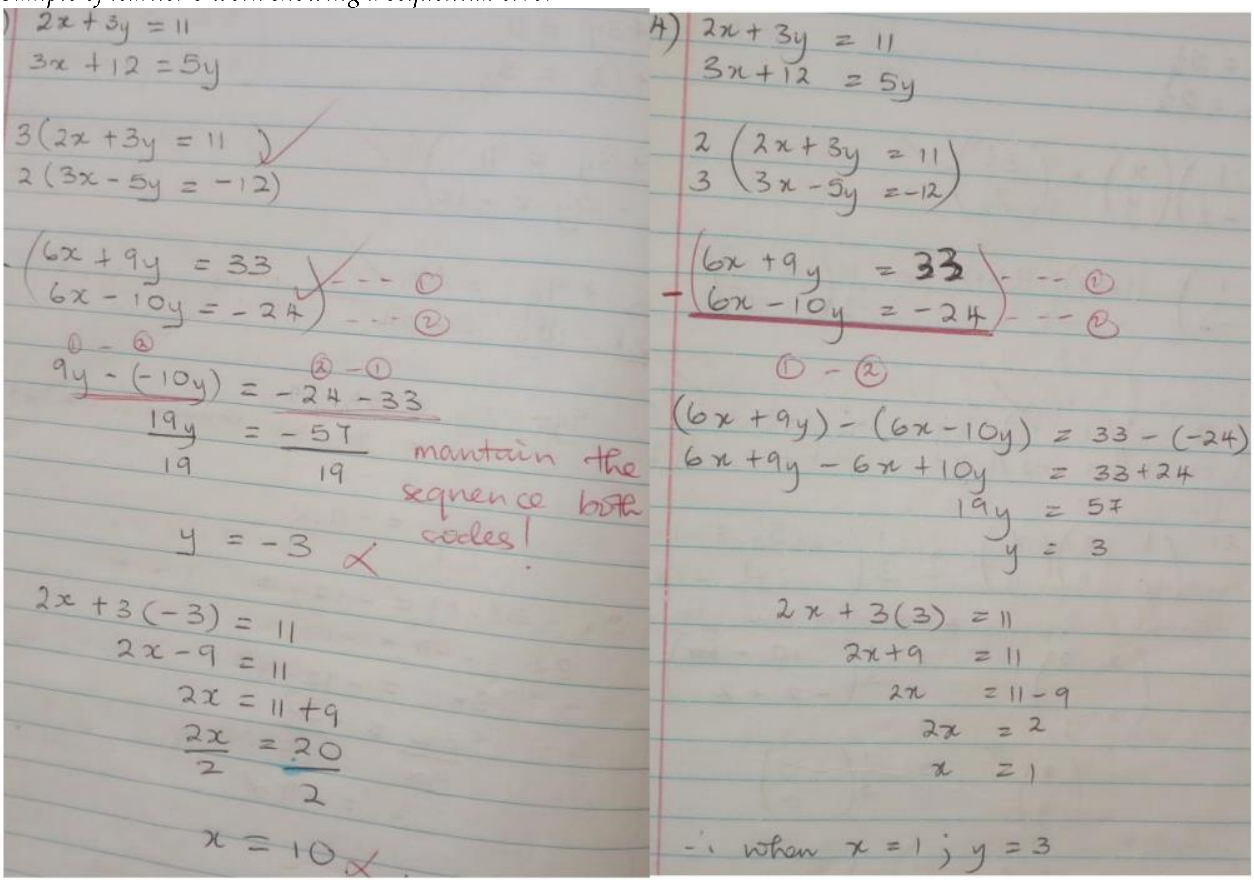
4.1.7. Sequential error (SE) and failure to respect sign

Another learner displayed sequential misunderstanding when solving Test Item 4 as shown in Figure 7 below. On the left-hand side, the equations were sequenced correctly, but not on the right-hand side. Instead of $33 - (-24)$, as if it is not possible to subtract a negative number.

Data revealed that when using the elimination method, learners would correctly choose the sign but failed to pay attention to the sequence and uniformity. They wrote the correct sign but failed to use it or reverse the sequence of simplification.

In answering Item 4 ($2x + 3y = 11, 3x + 12 = 5y$), one learner rearranged well and multiplied each equation to get $6x + 9y = 33$ and $6x - 10y = -24$, respectively. Also, a correct sign was chosen ($-$) but in executing the elimination, the sequence was ignored, as shown in Figure 7 below.

Figure 7
Sample of learner's work showing a sequential error



Furthermore, interviews revealed underlying misconceptions as the learners tried to explain challenges with subtracting big numbers. The error was finding differences between positive numbers and negative numbers.

> Researcher: ummmmmm, by the way, if we label equation 1 and 2, how did you subtract the equations?
> Learner: I subtracted equations 2 from 1.
> Researcher: Aha, in the elimination you subtracted the equations differently on both sides. Is there a problem with that?
> Learner: ummm sorry, I was supposed to subtract equation 2 from 1 on both sides too.
> Researcher: What exactly would you want be covered as prior knowledge?
> Learner: I think basic subtraction of bigger numbers is very important and can be used here

Some learners make mistakes when subtracting or adding two equations in order to eliminate one unknown; simply failing to follow chosen sequence. In Figure 7 above, the learner had chosen $3x + 1.5y = 3$

(i) and $3x - 4y = 14$ (ii). On the left-hand side our learner subtracted equation (i) from (ii) but on the right-hand side, the same learner chose to subtract equation (ii) from (i), resulting in a sequential error.

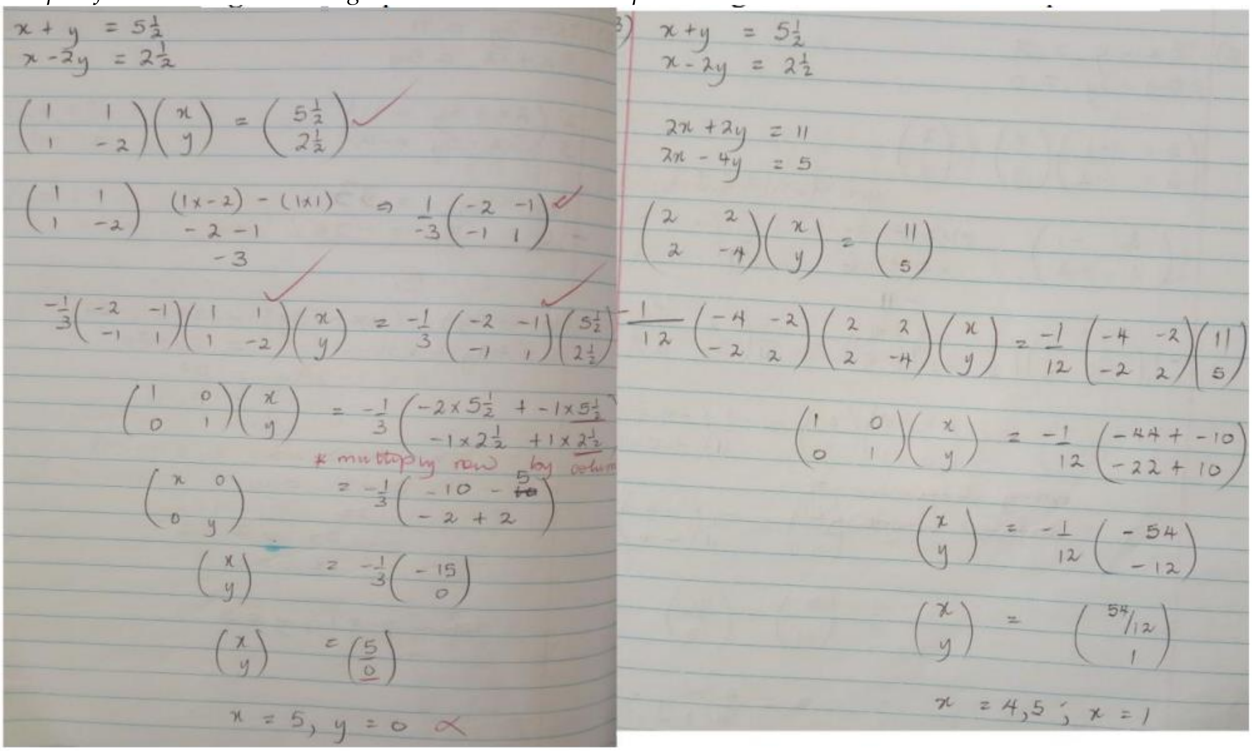
4.1.8. Wrong multiplication (WM) of matrices

The nature of the question from item 4, $(2x + 3y = 11,\ 3x + 12 = 5y)$, is such that the inverse exists; hence the matrix method can also be applied to solve it. Some learners displayed Wrong Multiplication (WM) of matrices as shown in Figure 8. Our study has shown that when the subject is made into a complicated fraction format, there are risks of making mistakes. We concur with Low et al. (2020) who reported that fractions left uncleared (not simplified) could lead to errors, when using matrices to solve simultaneous linear equations.

Another error noted is the inability of the participant to correctly apply row by column concept, as shown in Figure 8 below. The learner multiplied the row with the same figure from the second matrix instead of taking the whole column from the second matrix.

Figure 8

Sample of learner's work showing an error on matrix multiplication



4.2. Prerequisite Knowledge for Solving of Simultaneous Linear Equations

Document analysis of the responses on the assessment test scripts and the follow up interviews allowed the researchers to determine four categories of prerequisite knowledge required by learners to develop the full scope of understanding, namely; directed numbers, basic algebra, matrices, critical thinking, and fractions.

First, data revealed that directed numbers are a prerequisite as evident in the two excerpts below;

> Researcher: Do you think revising other previous topics which are required is essential, if yes, which are supposed to be done?
>
> Learner: umm yes, generally consistent practice is needed altogether. Directed numbers, basic algebraic processes were important for me and needs to be covered.

Yet, another learner had this to say

> Researcher: ooh, is there another sign for numbers besides negative and positive? Can you draw and label the number line and pick which 6 we are referring to?
>
> Learner: umm I cannot manage; the number line here is not given

The two conversations reveal that the learner did not realise the importance of the number line and the directed numbers in basic algebra activities.

Other learners were explicit that they required revising and finding the differences between positive and negative numbers. See conversation below:

> Researcher: What exactly would you want to be covered as prior knowledge?
> Learner: I think basic subtraction of bigger numbers is very important and can be used here

Directed numbers have both magnitude and direction (positive or negative). Learners must remember two rules; if the signs are the same, one must add the numbers and keep the original sign. If the signs are different, one must subtract and keep the sign of the bigger number.

Second, conversations revealed that basic algebra was are essential for solving simultaneous linear equations.

> Researcher: Which topic do you think should have been done before the lesson on simultaneous linear equations?
> Learner: umm sir I don't remember well the correct name, but I think basic equations must be revisited.

Another learner asserted that, "I think the topic 'expansion and removing of brackets' would be helpful where I need to substitute in an equation with brackets."

Basic algebra involves symbols and the rules for manipulating those symbols, that is, basic arithmetic operations (addition, subtraction, multiplication, and division within the algebraic expressions). Three rules apply. According to the commutative rule, we 'commute' when we add or multiply, but cannot 'change' or 'switch' the order of the numbers when we subtract. In the associative law, the parentheses move but the numbers or letters do not. The associative law works when we add or multiply and NOT when we subtract or divide. The distributive law says 'multiply everything inside the parentheses by what is outside it'. When we expand an algebraic expression, we use the distributive law to remove brackets and then we collect like terms.

Third, that knowledge and understanding of matrices is a prerequisite to using the matrix method in solving simultaneous equations. To use the matrices method, a learner must be able to multiply matrices.

> Researcher: So, what do you need to be covered as prior knowledge which you think may be helpful in solving simultaneous linear equations?
> Learner: Basically, I am more comfortable with matrices but I think the issue of determinant and inverse of a matrix needs to be more precise and be taught in such a manner that I won't forget or be confused again and my knowledge of the directed numbers needs to be sharpened as well.

In another interview, the prerequisite knowledge of multiplying matrices was evident.

> Researcher: by the way, do you know the order of the matrices you were multiplying?
> Learner: yes, it was a 2 by 2 multiplying a 2 by 1
> Researcher: ok, how did you determine the order of the resultant matrix?
> Learner: yes, its 2 rows and 1 column
> Researcher: Aha, you have indicated that on the right side, the row is multiplied by the corresponding term on the other matrix
> Learner: umm, yes, all the corresponding terms from both matrices can be simplified.
> Researcher: but that's for addition and subtraction ka.
> Learner: ooh sorry, I could not remember all the principles of multiplication. I think matrices should be redone.

Fourth, learners require prerequisite knowledge of critical thinking and multiplication of fractions.

> Researcher: Do you have any particular concept or topic which you wanted to be revisited or covered prior to simultaneous linear equations?
> Learner: Yes, iiih fractions are very important to me; avoiding them was part of knowing I have difficulties.

Learners require knowledge and understanding of how to determine the order of the resultant matrix and be able to multiply matrices. Similarly, it has been argued that learners need to develop more profound engagement with object conception and skills, which lead to schema development (Kazunga & Bansilal, 2020).

5. Conclusion

We concluded that learners encountered eight categories of errors when solving simultaneous linear equations, namely; Irrational Error (IE), Mathematical Error (ME), Inverse and Determinant error (ID), Complicating the Subject (CS), Wrong Substitution (WS), Wrong Choice of signs (WC), Sequential Error (SE), Wrong Multiplication of matrices (WM). Some of these manifest but as catalysts for subsequent errors.

Our second conclusion was that to solve simultaneous linear equations learners, require prior knowledge of directed numbers, basic algebra, matrices, critical thinking, and fractions in algebraic expressions.

Students come to the class with prior knowledge comprising of beliefs, ideas, and mathematical misconceptions that they have developed over the years in their previous classrooms, and life experiences (Naseer, 2015). Therefore, to develop a solid foundation and address prior knowledge and misconceptions, it is essential that the mathematics teachers must distinguish these common errors and misconceptions and provoke learners to reflect, revise and reconstruct these mathematical concepts. Teachers' can utilize their knowledge of errors and misconceptions to inform instructional decisions (Banerjee & Subramanian, 2012; Msomi & Bansilal, 2022; Welder, 2012).

6. Recommendations

Solving simultaneous linear equations is more technical than conceptual. Therefore, an appropriate instrument is needed to study misconceptions in solving simultaneous linear equations. We recommend further research using larger population and sample, as well exploring gender differences in errors and misconceptions displayed by students. The graphical method of solving simultaneous could be explored by including Test item that require use of the method in answering the question.

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