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Research Article

From student difficulties to design of a hypothetical learning trajectory: The case of system of linear equations in two variables

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This research aims to design a hypothetical learning trajectory for junior high school students in the case of systems of linear equations in two variables. The hypothetical learning trajectory is primarily designed according to the Realistic Mathematics Education (RME) theory to address students' difficulties. To do this, we carried out a two-stage qualitative case study. First, we investigated difficulties of 19 junior high school students (age 13-14 year-olds) when dealing with solving systems of equations in two variables and their application through an individual written test and interviews. Second, based on the first stage and the RME theory, we designed a hypothetical learning trajectory for the case of systems of linear equations in two variables. The results revealed that the main difficulties of the participants include, among others, transforming word problems into an appropriate mathematical model in the form of systems of linear equations of two variables, and making calculation or algebraic manipulation errors. Also, a hypothetical learning trajectory was designed, which consists of a learning goal, activities of the learning and teaching process, and predictions of student thinking and learning processes. In particular, we propose the use of the model method for overcoming difficulties in dealing with word problems. We conclude that the investigation of student difficulties provides fruitful information in the design process of hypothetical learning trajectory.

Keywords: Hypothetical learning trajectory; Junior high school students; Model method; Realistic mathematics education approach; Systems of linear equations in two variables

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1. Introduction

Understanding how students learn algebra has long been a core focus in mathematics education, especially when it comes to systems of linear equations in two variables (Irfan et al., 2019; Jupri et al., 2024). This topic demands that students coordinate multiple representations, think abstractly, and reason about relationships between quantities and symbols. Decades of research consistently show that many students find algebra challenging: they struggle to interpret variables, make sense of symbolic expressions, and see meaning behind algebraic procedures (Cañadas et al., 2018; Gupta & Zheng, 2020; Jupri et al., 2014). Much of this difficulty stems from a tendency to view algebraic symbols as isolated objects to be manipulated rather than as meaningful representations of relationships (Erbilgin & Gningue, 2023; Liu et al., 2019; Radford, 2022). These insights point to the importance of instructional approaches that build conceptual understanding and help students connect procedures to the underlying mathematical ideas.

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For junior high school students, systems of linear equations in two variables present an added layer of complexity because they require the ability to move fluently among algebraic, graphical, and contextual representations (Jupri et al., 2020). Students commonly face challenges when graphing equations, translating word problems into systems, or recognizing that a solution must satisfy both equations simultaneously (Fuchs et al., 2021; Jupri et al., 2024; Kaur, 2019; Verschaffel et al., 2020). When the tasks are purely symbolic, many rely on mechanical substitution or elimination without understanding why these methods work (Coles & Sinclair, 2019). Such patterns highlight the need for instructional sequences that guide students from intuitive, context-based strategies toward more formal algebraic methods.

The idea of a hypothetical learning trajectory [HLT] provides a useful way to address these issues. Originating in constructivist perspectives on learning design, an HLT includes three key components: the intended learning goals, the learning activities planned to achieve them, and the anticipated journey students may take while engaging with those activities (Bakker, 2018; Moss & Lamberg, 2019; Simon, 1995). By anticipating how students think and how their ideas might develop, an HLT allows teachers and researchers to design lessons that are responsive to students' initial conceptions and likely challenges. In algebra, HLTs have been applied to understand how learners progress from informal reasoning about quantities to formal algebraic expressions manipulation (Baroody et al., 2022; Doorman et al., 2012).

Realistic Mathematics Education [RME], a well-established instructional theory from the Netherlands, provides an important foundation for crafting such trajectories. RME emphasizes starting from meaningful, real-world contexts so that students can build mathematical ideas through progressive formalization (Fredriksen, 2021; Gravemeijer, 1994; Van den Heuvel-Panhuizen & Drijvers, 2020). A core principle within RME is guided reinvention, the idea that students should be supported in "reinventing" mathematical concepts through problem solving, reflection, and modeling. In the context of systems of linear equations, RME-based approaches often begin with realistic situations involving co-varying quantities—such as comparing prices, mixtures, or rates—before introducing equations and graphs as more formal models.

Another important RME principle relevant to this study is the development of models—what students initially construct as a model of a situation gradually becomes a model for more general mathematical reasoning (Gravemeijer & Doorman, 1999; Van den Heuvel-Panhuizen & Drijvers, 2020). Students might begin with tables, sketches, or informal notations to represent relationships in a context. Over time, these representations evolve, helping them recognize linear relationships and eventually understand the idea of solving two equations simultaneously. This natural progression aligns closely with the design of an HLT, which aims to anticipate when and how learners' informal models can develop into formal algebraic structures.

Bringing together students' common difficulties, RME principles, and the HLT framework creates a coherent foundation for designing effective instruction on systems of linear equations in two variables. Context-based tasks, opportunities for informal reasoning, and deliberate transitions toward formal methods all help students move beyond misconceptions and develop deeper understanding. A well-designed HLT also makes explicit the expected progression of student thinking, enabling researchers to test and refine their instructional designs through classroom observations (Bakker, 2018; Bakker & Smith, 2016; Gravemeijer & Cobb, 2006).

Building on these ideas, the present study aims to design a hypothetical learning trajectory for teaching systems of linear equations in two variables, informed both by documented student difficulties and by the theoretical foundations of Realistic Mathematics Education. By examining how students work with contextual, graphical, and symbolic tasks—and how their thinking develops across instructional stages—this study contributes to both practical teaching materials and the broader theoretical conversation on how students learn algebra. Ultimately, the goal is to bridge the gap between research on student difficulties and the creation of instructional approaches that meaningfully support and improve student learning.

3. Method

3.1. Research Design

To develop a hypothetical learning trajectory for junior high school students learning systems of linear equations in two variables, we designed a qualitative case study conducted in two main stages (Yin, 2016). In the first stage, we explored the difficulties experienced by 19 junior high school students as they solved systems of linear equations and applied them in contextual situations through and individual written test and interviews. In the second stage, insights from this analysis, together with principles from Realistic Mathematics Education, were used to design the proposed HLT.

3.2. Participants

The study involved 19 junior high school students aged 13–14 from a private school in Bandung, Indonesia. These students had previously learned foundational algebra topics, including linear equations in one variable, proportions, and operations with algebraic expressions. Given this background, we considered them to have sufficient prior knowledge and skills to take part meaningfully in the study.

3.3. Data Collection

During the first stage, we collected data through an individual written test on solving systems of linear equations. The test, completed by all 19 participants, was designed to capture how students approached the problems in a controlled environment. Each student was given 80 minutes to finish all seven tasks on systems of linear equations in two variables, and the use of smartphones or other electronic devices was prohibited to ensure that all responses reflected students' own thinking. We also collected their scratch work, which offered valuable insights into the reasoning, strategies, and step-by-step decision-making they used throughout the problem-solving process. Together, the final answers and scratch work provided a rich basis for analyzing both outcomes and underlying thought. In addition, we conducted interviews with five students to better understand why they chose particular solution strategies for solving systems of equations and to clarify parts of their written work that were unclear. These five students were selected because their test responses reflected key solution approaches and common difficulties. The interviews were guided by a structured interview protocol, which included non-intervening questions aimed at clarifying their reasoning and exploring the reasons behind their chosen strategies without influencing their responses.

3.4. Data Analysis

Data analysis for seven tasks on solving systems of linear equations in two variables was conducted in two stages. In the first stage, each task was checked for correctness. Each task was assessed according to a prepared rubric, with a maximum score of 10.

In the second stage, data analysis was conducted to determine students' difficulties in solving the tasks at each step of the problem-solving process. The data analysis process for this second stage proceeded as follows. We first reviewed students' written work to identify the specific difficulties they faced when working with systems of linear equations in two variables. This included challenges in symbolic manipulation as well as difficulties in interpreting and solving word problems that served as real-life applications of the topic. We examined not only the correctness of their answers but also the procedures they followed, paying close attention to their intermediate steps and notes. This scrutiny allowed us to trace how students approached each task, the strategies they employed, and the points where misunderstandings or breakdowns occurred. From this, we categorized the types of difficulties, such as misunderstandings about variable relationships, procedural or calculation errors, and challenges in translating contexts into equations. After identifying these key difficulties, we combined the findings with theoretical insights from RME. This synthesis guided the development of the hypothetical learning trajectory, ensuring that the resulting HLT addressed students' needs, supported their conceptual growth, and helped them progress toward more formal and effective strategies for solving systems of linear equations.

4. Results and Discussion

This section presents the study's findings and discussion across two stages. First, we describe the difficulties students faced when working with systems of linear equations in two variables. Second, we explain how these findings informed the development of a hypothetical learning trajectory to support students' understanding of the topic.

4.1. Student Difficulties in dealing with System of Linear Equations

Table 1 presents the results of investigation on student difficulties in solving system of linear equations in two variables. Task 1 and Task 2 assess student ability in solving systems of equation symbolically. Tasks 3 to 7 assess student ability in solving word problems as the application of the concepts of system of equations.

Table 1 shows that only about half of the students were able to correctly solve Tasks 1 and 2, which focused on systems of linear equations in two variables. Their performance dropped even further on Tasks 3 to 7, where only around 20 percent successfully solved the word problems. These results suggest that while some students can manage symbolic equations, far fewer can interpret and translate real-world situations into mathematical models. This gap indicates deeper conceptual challenges.

Table 1
Results of written test of systems of linear equations in two variables (N = 19)

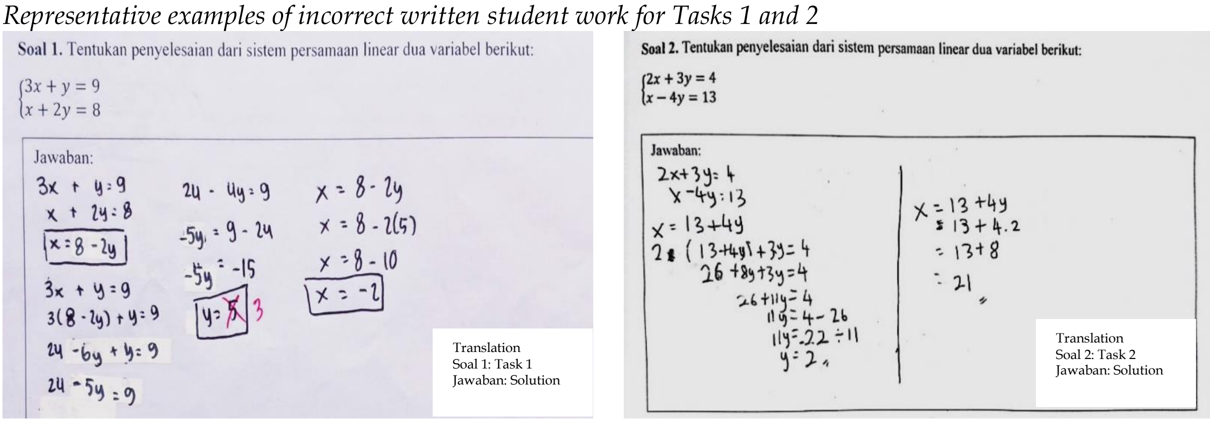
Tasks	*# Correct (%)*	*Type of errors and difficulties*
Task 1. Find solution for the following system linear equations in variables. $\begin{cases}3x + y = 9\\x + 2y = 8\end{cases}$	9 (47.3)	- Making calculation errors - Forgetting methods of solving a system of equations - Forgetting the prerequisite of systems of linear equations - Not knowing how to check whether a solution of a system of equations is correct or not.
Task 2. Find solution for the following system linear equations in variables. $\begin{cases}2x + 3y = 4\\x - 4y = 13\end{cases}$	12 (63.1)	- Making calculation errors - Forgetting methods of solving a system of equations - Forgetting the prerequisite of systems of linear equations - Not knowing how to check whether a solution of a system of equations is correct or not.
Task 3. Given the sum of two numbers is 51. While the difference between the two numbers is 29. Determine each of these numbers.	4 (21.1)	- Failing to make an appropriate mathematical model (system of equations) from the word problem - Forgetting methods of solving a system of equations - Making calculation errors - Not knowing how to check whether a solution of a system of equations is correct or not.
Task 4. The price of a toothbrush and a toothpaste is Rp28,500. The toothpaste is Rp1,500 more expensive than the toothbrush. Find the price of each item.	4 (21, 1)	- Failing to make an appropriate mathematical model (system of equations) from the word problem - Forgetting methods of solving a system of equations - Making calculation errors - Not knowing how to check whether a solution of a system of equations is correct or not.
Task 5. Tom bought two notebooks and one pencil for a total of Rp25,000. At the same store, Yeni bought three notebooks and two pencils for a total of Rp42,500. How much did each notebook and pencil cost at the store?	9 (47.3)	- Failing to make an appropriate mathematical model (system of equations) from the word problem - Forgetting methods of solving a system of equations - Making calculation errors - Not knowing how to check whether a solution of a system of equations is correct or not.
Task 6. Three years from now, my father's age will be three times Budi's age. Three years ago, my father's age was five times Budi's age. How old are Budi's father and Budi's age now?	2 (10.5)	- Failing to make an appropriate mathematical model (system of equations) from the word problem - Forgetting methods of solving a system of equations - Making calculation errors - Not knowing how to check whether a solution of a system of equations is correct or not.
Task 7. Mr. Udin keeps a number of chickens and goats. The number of chickens and goats that Mr. Udin has is 25. Meanwhile, the number of chicken legs and goat legs is 64. Determine the number of each chicken and goat that Mr. Udin has.	4(21.1)	- Failing to make an appropriate mathematical model (system of equations) from the word problem - Forgetting methods of solving a system of equations - Making calculation errors - Not knowing how to check whether a solution of a system of equations is correct or not.

Student difficulties that can be investigated from Tasks 1 and 2, as symbolic problems for the system of linear equations in two variables, include making calculation errors, forgetting the methods of solving a system of equations, forgetting the prerequisite knowledge of systems of linear equations, and not knowing how to check whether a solution of a system of equations is correct or not. From the RME theory, these findings suggest that the students encountered difficulties in dealing with the symbolic world of situations, which is called vertical mathematization difficulties (Doorman et al., 2007; Glade & Prediger, 2017; Jupri & Drijvers, 2016). This interpretation is consistent with other studies indicating that students often struggle with core algebraic concepts, that a meaningful transition from arithmetic to algebra requires careful attention to concepts such as variable, equality, and equation, and that students may rely on memorized or procedural approaches rather than algebraic generalization (Bozkuş et al., 2025; Santoso et al., 2019; Tanışlı et al., 2020; Yıldız & Yetkin Özdemir, 2021). To illustrate the findings, we provide examples of representative written student solution and its corresponding interview results. Figure 1 is representative examples of incorrect written student work in solving Task 1 and Task 2, respectively. Figure 1(a) shows an example on how a calculation error occurred for the case of Task 1. Rather than getting $y = 3$ from $-5y = -15$, the student concluded $y = 5$ incorrectly. The next step of getting the value of x is incorrect as the consequence of the incorrect value of y.

Similar to the case of Figure 1 (a), it can be seen in Figure 1(b) that the student is using a substitution method to solve the system of equation for the case of Task 2. One the one hand, the substitution procedure can be carried out correctly, but she made a calculation error when solving the equation $11y = -22$. Rather than getting $y = -2$, she concluded $y = 2$. Next, based on this, she got $x = 21$. This solution process suggests that the student is able to use the substitution method, fails to solve a linear equation in one variable, and did not check if her solution correct or not, due to a miscalculation in one of the steps of solution process. When she was interviewed to clarify the solution process, she replied that "*Even if I am a bit forget, I can solve the task using the substitution method*." Next, when she reminded that her solution is incorrect, she said "*Oh, I see that I made a calculation error in solving* $11y = -22$."

Figure 1

Representative examples of incorrect written student work for Tasks 1 and 2



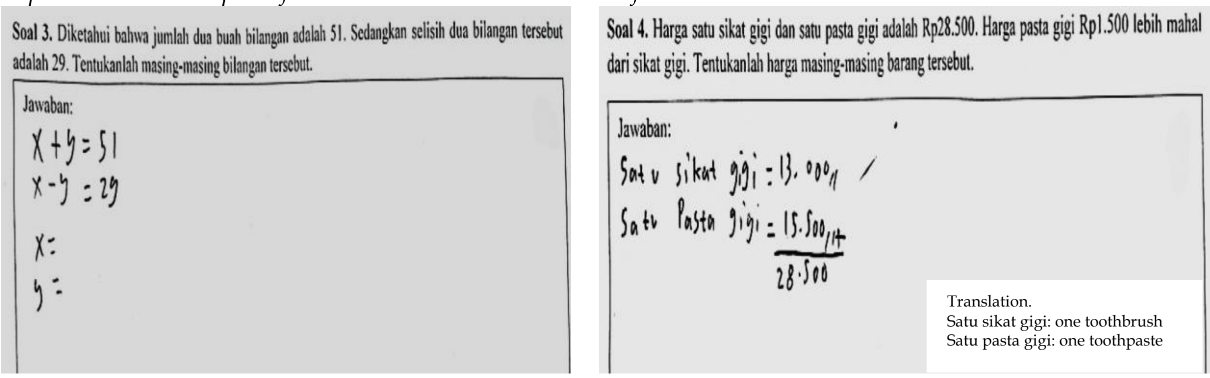
(a) (b)

Tasks 3 to 7 concern word problems that are the application of the system of linear equations in two variables. Student difficulties that can be investigated from Tasks 3 to 7 include failing to make an appropriate mathematical model (system of equations) from the word problem, forgetting methods of solving a system of equations, making calculation errors, and not knowing how to check whether a solution of a system of equations is correct or not. From the RME perspective, the findings on solving word problems suggest that the students mainly encountered difficulties in dealing with translating from the world of situations to the world of symbols, which is called horizontal mathematization difficulties (Doorman et al., 2007; Jupri & Drivers, 2016; Van den Heuvel-Panhuizen, 2019). The findings related to word problems are consistent with previous research showing that students often struggle to identify relevant relationships in a context, formulate appropriate mathematical models, and move flexibly between verbal and symbolic forms (Daroczy et al., 2015; Jupri & Drijvers, 2016; Mingke & Alegre, 2019). These difficulties indicate that the challenge lies not only in performing algebraic procedures but also in making sense of the problem situation and expressing it mathematically in a meaningful way. To illustrate the findings of student difficulties in dealing with word problems, we provide representative written student solution and its corresponding interview results in dealing with word problems for the case of Task 3, 4, 5, and 6. The case of Task 7 is not illustrated because it is similar to the case of these tasks.

Figure 2 (a) is an example of written student work in dealing with Task 3. The student is able to translate the text into a correct mathematical model in the form of a system of equations, but forget on how to solve the system. Figure 2(b) is an example of written student work in dealing with Task 4, in which he was not able to construct a mathematical model.

Figure 2

Representative examples of incorrect written student work for Tasks 3 and 4



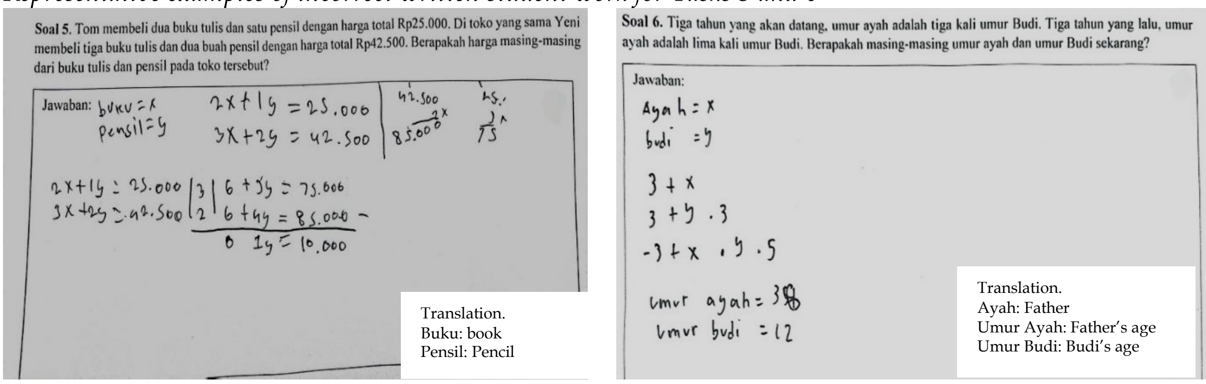
(a) (b)

Figure 3 (a) is an example of written student work in dealing with Task 5. The student is able to translate the text into a correct mathematical model in the form of a system of equations, is able to eliminate the variable x to get $y = 10{,}000$, but is not able to continue the work to get the value of x. Figure 3(b) is an example of written student work in dealing with Task 6, in which he was not able to construct a mathematical model.

From Figure 3(b), it can be seen that the student encountered difficulties in constructing a mathematical model in the form of system of equations. This starts from a difficulty in translating the first sentence *"Three years from now, my father's age will be three times Budi's age"*. Rather than translating it into $(x + 3) = 3(y + 3)$, the student wrote $(3 + x)$ and $3 + y.3$. A similar case occurred for the case of the second sentence which needs to translate the sentence "Three years ago, my father's age was five times Budi's age." into $(x - 3) = 5(y - 3)$. In the interview, the student mentioned that *"I am very confused on how to translate the first and second sentence into equations."*.

Figure 3

Representative examples of incorrect written student work for Tasks 5 and 6



(a) (b)

4.2. Hypothetical Learning Trajectory for the Learning of System of Linear Equations

Drawing on the findings presented in the previous section and guided by the principles of Realistic Mathematics Education, we developed a hypothetical learning trajectory for introducing systems of linear equations in two variables to junior high school students. The HLT includes three key components: the learning goals, the planned learning and teaching activities, and predictions about how students are expected to develop their understanding throughout the lessons (Simon, 1995; 2020). The designed HLT, which is still theoretical and has not been tried in practice, is described in detail in the following paragraphs.

The overall goal for the learning of systems of equations is "to understand what it means to solve a system of linear equations and can flexibly use informal and formal strategies to find the solution of the system." This goal is broken down into three main activities, i.e., informal reasoning in contextual problems, transition to symbolic representation, and solving systems of equations using substitution, elimination, and graphical methods. These activities and the prediction of students' thinking processes of learning are described below.

4.2.1. Activity 1: Informal reasoning in context

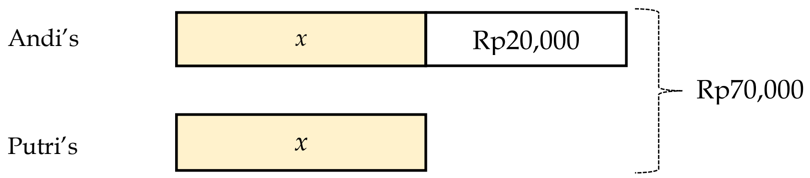
The learning trajectory begins with meaningful, real-life contexts that invite students to reason informally (Laurens et al., 2017; Van den Heuvel-Panhuizen & Drijvers, 2020). For instance, students might compare two different price combinations. The first combination includes two glasses and one shorts for Rp760,000, while the second offers one glass and two pairs of shorts for Rp800,000. At this stage, students often experiment with guess-and-check, make simple tables, or look for patterns in the prices. Their goal is to figure out when the two combinations would cost the same. As they explore, students start noticing how the quantities change together—a sense of co-variation—and eventually identify the point where the costs become equal. They naturally interpret this equality as the solution to the problem. In line with other studies (Jupri & Drijvers, 2016; Van Zanten & Van den Heuvel-Panhuizen, 2021), through this activity students are expected to be able to construct a system of equations from real-world problems, and to solve it using informal strategies.

4.2.2. Activity 2: Transition to symbolic representation

In this phase, students begin moving from contextual or table-based reasoning toward more symbolic representations. They express the prices of the glasses and shorts as a system of equations, such as $2g + s = 760{,}000$ and $g + 2s = 800{,}000$, where g represents the price of a glass and s the price of a pair of shorts. At this point, students start to generalize the patterns they discovered earlier and understand that solving a system means finding values that make both equations true at the same time. Through these observations, they may notice that subtracting or combining information leads to relationships like $3s = 840{,}000$, giving $s = 280{,}000$. Similarly, they may identify $3g = 720{,}000$, which results in $g = 240{,}000$. This transition reflects students' growing ability to connect informal reasoning with symbolic methods.

Transition to symbolic representation can also be bridged using a model method. This method is primarily developed in Singapore since 1980s (Kaur, 2019; Ng & Lee, 2009). Using this method, word problems are represented in the form of rectangles. A rectangle represents a quantity or a relationship between quantities. Based on this representation, it is relatively easy to determine the mathematical model that needs to be solved. The model method is used to solve word problems involving addition, subtraction, multiplication, division, fractions, comparison, percentages, and algebraic operations. For example, let us solve the following word problem *"The total of Andi and Putri's money is Rp70,000. Andi has Rp20,000 more money than Putri. Determine how much money each of them has?"* The solution using this model method is described in Figure 4.

Figure 4
A solution of a word problem using the model method



Based on the representation, we can write

$$x + 20000 + x = 70000$$
$$\Leftrightarrow 2x + 20000 = 70000$$
$$\Leftrightarrow 2x = 50000$$
$$\Leftrightarrow x = 25000$$

So, Putri's money is Rp25,000 and Andi's money is Rp25,000 + Rp20,000 = Rp45,000.

4.2.3. Activity 3: Solving systems of equations using substitution, elimination, and graphical methods

In this phase, students are reminded of systems of equations obtained in the previous activity, namely $\begin{cases} 2g + s = 760{,}000 \\ g + 2s = 800{,}000 \end{cases}$. Next, students are introduced on how to solve this system symbolically using an elimination method, substitution method, the combination of elimination and substitution method, and the graphical method. In this way, students work within the world of symbolic mathematics.

5. Conclusion

This study aims to design a hypothetical learning trajectory to support junior high school students in learning systems of linear equations in two variables. Guided by the principles of Realistic Mathematics Education, the study followed a deliberate process: first identifying students' actual difficulties, and then using those findings to shape the components of the instructional design. This approach highlights the importance of grounding pedagogical decisions in real classroom evidence rather than relying solely on theoretical assumptions. Through an analysis of students' written test responses and follow-up interviews, several student difficulties emerged. Many students struggled to translate word problems into appropriate mathematical models, while others faced challenges in carrying out algebraic manipulations and calculations accurately. Together, these issues reveal meaningful gaps not only in students' procedural fluency but also in their deeper conceptual understanding of the topic.

Building on these insights, we developed an HLT that includes clearly defined learning goals, a logical sequence of learning and teaching activities, and predictions about how students' thinking might evolve throughout the lessons. A key strength of the proposed trajectory is its integration of informal strategies, the model method, and formal algebraic techniques. This progression is intentionally designed to help students move from intuitive reasoning, which is often grounded in real contexts, toward more formal symbolic representations and standard solution methods. As such, the HLT offers a coherent and supportive pathway for developing both understanding and skill. Overall, the findings reinforce that examining student difficulties is not only helpful but essential for crafting effective instructional trajectories.

Despite these promising outcomes, the present study is not without limitations. The research involved a small number of students from a single private school, which restricts the broader applicability of the findings. Moreover, while the HLT was carefully designed, it was not implemented in full classroom practice, leaving its practical effectiveness untested. Future research should therefore involve classroom-based trials to examine how students interact with the proposed activities and whether the trajectory meaningfully improves learning outcomes. Further studies might also investigate how technology, collaborative learning, or differentiated instruction can enrich students' understanding of systems of equations. Including a more diverse set of schools and students would also help strengthen the generalizability of future results.

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Data availability: Data supporting the findings and conclusions are available upon request from the corresponding author.

Declaration of interest: The authors declared that there were no potential conflicts of interest.

Ethical statement: This study, which involved 19 junior high school students, has been conducted according to the academic ethics that adhere to the rule of the Universitas Pendidikan Indonesia. Approval number: B-3915/UN40.A4.1/PK.02.02/2025.

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