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Research Article

Assessing the readiness of pre-service mathematics teachers to learn Euclidean geometry

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The introduction of the four year Bachelor of education mathematics curriculum in 2018 in the Ghanaian colleges of education brought back Euclidean geometry with its formal proof. It also implied that all learners needed to perform at level 4 (Deduction: with formal proof) of the van Hiele levels in Euclidean geometry in all grades in the CoE. This study reports on the assessment of the readiness of pre-service teachers' [PTs] to learn Euclidean geometry in Ghana based on the van Hiele model of geometrical thinking levels. Data for this study were generated from 302 PTs in a CoE in the volta region of Ghana selected through convenience sampling. The study made use of both quantitative and qualitative research techniques for data collection. The quantitative data revealed that a significant proportion of PTs demonstrated basic geometrical thinking, with a predominant presence at Level 1(Visualization) and Level 2(Analysis). Qualitative insights underscore a reliance on visual properties and informal language in identifying geometric figures. Among PTs, a moderate level of readiness was observed. The study suggests a need for targeted instructional interventions to enhance formal understanding of geometric figures among PTs and reinforces the importance of geometric terminology for improved readiness among PTs'. Acknowledging limitations and proposing future research directions, this study contributes valuable insights for geometry education in teacher preparation programs.

Keywords: Euclidean geometry, Geometry, Service teachers, van Hiele Geometry Test

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1. Introduction

Throughout the scholastic process, several facets of geometry are examined, and Ghana is not much different in this regard. The primary subject of this study is Euclidean geometry, and it places a high priority on pre-service teachers' [PTs'] readiness for it. Geometry has always been thought of as the measurement of the earth (Dillon, 2018). Geometry as we know is concerned with curves and surfaces of different dimensions. As a result, geometry has been divided into several branches, including differential geometry and algebraic geometry (Dillon, 2018). Euclidean geometry was the primary geometry topic taken into consideration in this study, both for research purposes and participant background descriptions. The foundation of Euclidean geometry in early mathematics consisted of definitions and postulates concerning shapes that were taken for granted. Certain definitions, which have endured over time, cover concepts such as lines, points, planes, angles, and so on (Singmaster, 1982). Drawing on the foundations established by the ancient Greek

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mathematician Euclid in his book "Elements," Euclidean geometry is a discipline of mathematics that studies geometric objects and their properties in two and three dimensions (Moglianesi & Harris, 2019).

Since Euclidean geometry is a crucial subject for mathematics instruction, it is included in Ghana's recently implemented four-year Bachelor of Education [B.Ed] mathematics curriculum (Ghana Education Service, 2019). The program aims to provide PTs with the necessary knowledge and abilities to effectively learn and instruct mathematics in Ghanaian basic schools. The Ghana Education Service (2019) claims that studying Euclidean geometry is crucial because it gives studentss a chance to hone their spatial reasoning abilities and problem-solving skills in the actual world. Euclidean geometry was incorporated into the Colleges of Education's new B.Ed. mathematics curriculum in Ghana, partly to help students become more adept at solving problems. According to Dowuona-Hammond (2015), Euclidean geometry gives students the chance to apply geometric concepts and principles to solve problems in the actual world. Furthermore, Euclidean geometry is an essential component of mathematics education, and including it in the curriculum supports the development of a well-rounded education for those who aspire to become mathematics educators (Adjei & Owusu-Sekyere, 2017). But according to Debrah and Baafi's (2018) study, Ghanaian mathematics PTs lacked the expertise needed to properly teach Euclidean geometry. In order to close this knowledge gap, the new B.Ed. mathematics curriculum gives students more in-depth instruction in Euclidean Geometry along with innovative teaching strategies. The new four-year B.Ed. mathematics curriculum implied that all mathematics pre -service teachers must perform fully at a higher level (level 4- Deduction: with formal proof) of the van Hiele levels in major aspects of Euclidean geometry. PTs are expected to "investigate line segments joining the midpoints of two sides of a triangle and properties of special quadrilaterals." Pre-service teachers are also expected to define, investigate, and make conjectures and prove conjectures on special quadrilaterals.

This paper reports on a study which made use of the van Hiele levels of geometric thinking to assess the readiness of PTs for Euclidean geometry. Display quotations of over 40 words, or as needed.

1.1. Statement of the Problem

In Ghana as well as other countries, Euclidean geometry is a common subject in classrooms. Euclidean geometry is included in the mathematics curriculum for both basic and secondary education in Ghana. Mathematics education should enable all students to learn important mathematical concepts and skills; develop a positive attitude towards mathematics; and be confident in their ability to use mathematics to solve problems, states the National Council of Teachers of Mathematics [NCTM] (2021). However, research indicates that PTs in Ghana frequently lack the pedagogical abilities and mathematical knowledge required to learn and teach mathematics, including Euclidean geometry (Akayuure et al., 2016). Furthermore, PTs' capacity to instruct Euclidean geometry can be impacted by Ghana's cultural background. Due to the country's multilingualism, pre-service teachers may find it difficult to properly explain mathematical concepts to students (Ankomah, 2020). Furthermore, pre-service teachers' capacity to instruct mathematics effectively can be impacted by societal attitudes toward education as well as financial limitations (Makara, 2019). Research on PTs' readiness to learn Euclidean geometry in Ghana is lacking. Although studies on mathematics instruction in Ghana exist, few of them concentrate on PTs' readiness to learn Euclidean geometry.

1.2. Purpose of the Study

The purpose of this study was to use the van Hiele levels of geometric thinking to assess the readiness of PTs to learn Euclidean geometry, with a particular focus on their understanding of foundational concepts and overall confidence in teaching the subject. Based on this purpose, the following objectives were proposed: 1) to determine the level of van Hiele geometrical thinking of mathematics student teachers in Peki college of education, 2) to find out the readiness of mathematics student teachers' to take Euclidean geometry course in the new four year B.Ed. mathematics curriculum. In pursuance of this objective, this research questions below were formulated to guide the study.

RQ 1) What is the level of van Hiele geometrical thinking of mathematics PTs in Peki College of Education?

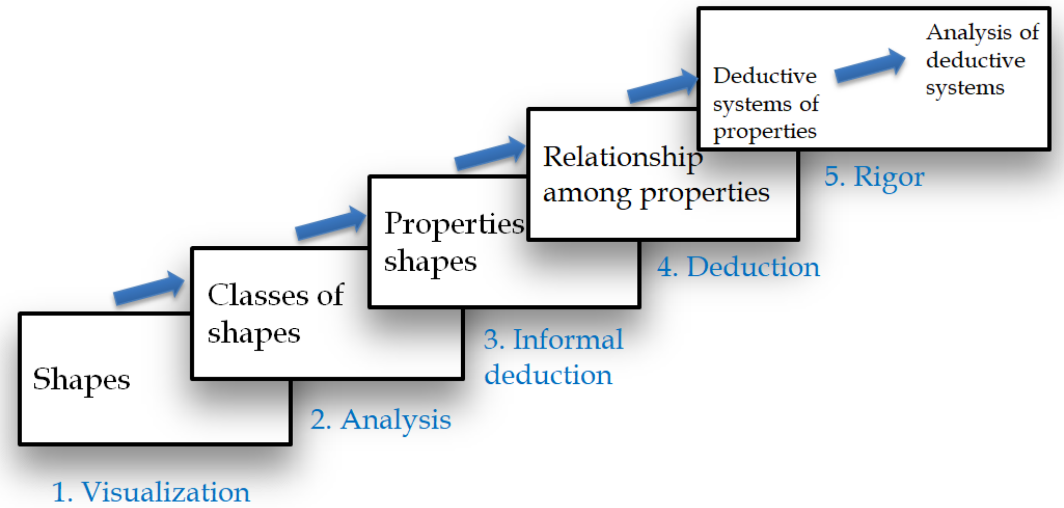
RQ 2) How ready are mathematics PTs in Peki College of Education to take Euclidean geometry course in the new four year B.Ed. mathematics curriculum?

2. Background

This study was hinged on the van Hiele theory as the theoretical framework for assessing PTs readiness to learn Euclidean geometry. The van Hiele theory of geometrical thinking was developed by Pierre van Hiele and his wife Dina van Hiele Geldof (Armah et al., 2017; Armah et al., 2018; Asemani et al., 2017) out of the frustrations the pair and their students experienced during teaching and learning of geometry. The two were Dutch researchers and mathematics teachers. The theory emanated from their thesis at the University of Utrecht in 1957 (Usiskin, 1982). According to van Hiele's theory, there are five levels of geometric thinking. These are as follows:

1. Visualization: learners recognize shapes and make observations about them.
2. Analysis: learners identify properties and relationships between shapes.
3. Abstraction: learners use symbols and diagrams to represent geometric concepts.
4. Deduction: learners make logical deductions based on the properties and relationships of geometric figures.
5. Rigor: learners are able to prove geometric theorems using logical reasoning (Alex & Mammen, 2016; Usiskin, 1982) in ascending order of difficulty (see Figure 1).

Figure 1
Van Hiele's Level of geometrical thinking



The theory has enumerated three parts as (i) the existence of levels, (ii) the properties of the levels, and (iii) the phases of learning from one level to the next level (Crowley, 1987; Vojkuvkova, 2012). Initially van Hiele theory was numbered from 0 to 4. However, (Alex & Mammen, 2016; Siew & Chong, 2014) studies have increased it to level 5. This modification permits a sixth level named pre-recognition for students who will not achieve van Hiele level 1 (Mason, 1998).

2.1. Readiness Assessment in Mathematics Education

Readiness in mathematics education is a multifaceted concept, encompassing cognitive, affective, and pedagogical dimensions. Researchers (Piaget, 1971; Vygotsky, 1978) highlight the importance of assessing not only students' knowledge but also their cognitive structures, beliefs, and attitudes. Previous studies (e.g., Ginsburg, 2009; Ma, 1999) have explored readiness in various mathematical domains. Ginsburg's (2009) research particularly highlights the need for a comprehensive assessment that goes beyond surface-level knowledge and delves into the cognitive structures that underpin mathematical understanding. These works emphasize the necessity of understanding learners' cognitive development and pre-existing knowledge as crucial factors in effective pedagogy. The application of the van Hiele Theory to readiness assessment is rooted in its recognition of the hierarchical nature of cognitive development in geometry (Fuys, 1988). Assessing readiness involves recognizing where individuals are situated within the van Hiele hierarchy and tailoring instructional approaches accordingly.

2.2. Studies on Van Hiele Theory in Ghana

The study by Abu et al. (2013) was conducted in five education colleges that specialize in mathematics and science in Ghana's three northern regions. The investigation of pre-service math teachers' van Hiele levels of geometric thinking was the aim of the study. It was determined that a mere 11.44% of Ghanaians were qualified to instruct in elementary school mathematics. His research revealed once more that 88.56% of PSTs in mathematics achieved levels 0 to 2, making them ineligible to instruct math in basic schools. Moreover, a study by Armah et al. (2017) concentrated on the van Hiele geometric thinking of PSTs in Ghana prior to their application in the classroom. 75.33% of PSTs were lower than their projected future JHS 3 students, according to their conclusion based on van Hiele geometry test levels. Furthermore, Asemani et al. (2017) chose 200 senior high school students from three (3) municipalities in the Central Region of Ghana to serve as their research subjects. The authors came to the conclusion that 43% of secondary school students in Ghana did not reach any van Hiele Geometric thinking level in their final year. Baffoe and Mereku (2010), reported their study by a sample size of 188 from Winneba Senior High School and Zion Girls schools. The sample students came from the Winneba metropolis. Baffoe & Mereku measured the van Hiele levels of geometric thinking of Ghana's Senior High School [SHS] 1 students. Their study was conducted when the students were four weeks old in the SHS campus level. The emanated results of the study indicated that Ghana's SHS 1 students were lagging behind on performance as compare to their colleagues from other countries when the van Hiele geometric thinking test analysis was done. The quantitative results analysis showed that 59%, 11%, 1% of Ghana's SHS 1 students attained van Hiele levels 1, 2 and 3 respectively.

2.3. Originality and Need for the Study

While previous studies (Abu et al., 2013; Armah et al., 2017; Asemani et al., 2017) have provided valuable insights into van Hiele geometric thinking in Ghana, a notable gap exists in the simultaneous examination of pre-service mathematics teachers (PTs) and secondary school students. Our study seeks to fill this void by adopting a comprehensive approach that integrates both perspectives. By examining the van Hiele levels among PTs and students concurrently, our study seeks to establish a more holistic understanding of the dynamics within the Ghanaian educational system. Furthermore, our methodological approach, which incorporates qualitative data collection from selected participants, adds a distinctive dimension to the study. This qualitative component allows us to explore nuanced aspects of geometric thinking and gain insights into the lived experiences and perceptions of those involved in the educational process. This unique focus contributes to the originality of our research and addresses the current limitations in the literature.

The insights gained from our research can contribute to the development of tailored training programs for PTs and targeted interventions for students, fostering an improvement in overall geometric understanding within the Ghanaian educational landscape. In conclusion, we believe that our study's originality lies in its integrated approach, combining a dual focus on PTs and students, and its qualitative component, which adds depth to the analysis. The need for the study is underscored by the identified gaps in the literature and the potential to catalyze positive change within the Ghanaian educational system.

3. Method

3.1. Research Design

The pragmatic paradigm and a mixed method approach—more especially, the sequential explanatory method design—were chosen as the study's methodology based on the research questions. A mixed method entails gathering both quantitative and qualitative data, combining the two sets, and deriving specific conclusions from them (Creswell, 2014). Because of the design's broad scope, a sequential explanatory method design can yield extensive and detailed data (Creswell & Plano Clark, 2018). According to Cohen et al. (2017), the quantitative and qualitative aspects of the methodological dimensions must be covered, making it essential that the methods be explicitly justified for both the quantitative and qualitative components. In essence, the structure of this design is such that the collection of quantitative and qualitative data is done independently. The analyzed data can be interpreted, but the discussions can be conducted so that the qualitative results support or refute the quantitative findings, and vice versa (Creswell & Plano Clark, 2018). The majority of the questions that were given to participants as quantitative premises were modified to include open-ended questions in order to create data diversity. Data diversity is strongly advised in order to evaluate the overall quality of mixed methods studies (Tashakkori et al., 2021). A balanced use of quantitative and qualitative data was intended to overcome the limitations of these data alone for confirming and strengthening the findings and providing a much more thorough explanation of the problem, as stated in the pertinent literature (Creswell, 2014).

3.2. Participants

The level 200, 300, and 400 mathematics students at Peki College of Education in Ghana's Volta Region made up the study's population, which sought to ascertain PTs' readiness to study Euclidean geometry within the framework of the country's new four-year B.Ed. mathematics curriculum. The term "level" under participants refers to the academic year or class level of the PTs mathematics students at Peki College of Education. The students are categorized into three groups based on their respective academic years: Level 200, Level 300, and Level 400. These designations correspond to the different stages of the four-year Bachelor of Education (B.Ed.) mathematics curriculum.

Level 200: Students in their second year of the B.Ed. mathematics program.

Level 300: Students in their third year of the B.Ed. mathematics program.

Level 400: Students in their fourth and final year of the B.Ed. mathematics program.

The determination of these levels is based on the standard academic structure of the college, where students progress through different levels as they advance in their studies. The selection of this college was influenced by a number of factors, including functionality, proximity, and geographic accessibility. 302 PTs were chosen at random from the total population using a convenient sampling technique. Time, money, and manpower were among the practical factors that the researchers took into account when choosing to sample 302 PTs'. It may have been logistically difficult and resource-intensive to administer surveys or assessments to more than the 302 participants.

Furthermore, six (6) participants were chosen using the purposive sampling technique in order to collect more information for the qualitative component. In qualitative research, purposive sampling involves selecting participants who possess specific characteristics or experiences relevant to the research questions, contributing rich and in-depth insights into the phenomenon under investigation. The participants were chosen based on their varying levels of van Hiele geometric thinking, as identified through the initial quantitative phase of the research. We aimed to include participants representing a spectrum of readiness levels to ensure a comprehensive understanding of PTs' readiness to study Euclidean geometry within the new B.Ed. mathematics curriculum. The selection also considered diverse perspectives and experiences among PTs to capture a range of insights. Upon reaching the sixth participant, we observed that thematic saturation had been achieved and the researchers concluded that no additional participants were required to provide a sufficient response to the research questions. The study's data collection was finished after the intended sample size was attained, and it took place during the second semester of the 2022/2023 academic year. A description of the PTs participating in the study is presented in Table 1.

Table 1
The demographics of the recruited study participants

Measure	*Number(n)*	*Percentage (%)*
Gender		
Male	174	58
Female	128	42
Age		
<20	98	32
21-30	156	52
31 and above	48	16
Level		
200	125	42
300	101	33
400	76	25

Table 1 shows that, 174(58%) of the 302 pre-service teachers were male and 128(42%) were female; according to the distribution of age ranges, 98(32%) were less than 20 years old, 156 (52%) were 21-30 years old and 48 (16%) were 31 years old or above; lastly according to the pre – service teachers level variable, 125(42%) were in level 200, 101(33%) were in level 300, and 76(25%) were in level 400.

3.3. Data Collection Instrument and Procedures

Based on the research questions, the data collection instruments used were the van Hiele Geometry Test [VHGT] items and structured interview (open ended tasks dealing with geometric shapes), developed by Burger and Shaughnessy (1986) adapted. The VHGT was used to collect the quantitative data and the structured interview (open ended tasks dealing with geometric shapes) were used to collect the qualitative data.

3.3.1. Van Hiele Geometry Test

Van Hiele's geometric thinking levels and the PT's Euclidean geometry achievement were compared using the VHGT. There were four subtests totaling twenty multiple-choice questions on the test. Each subtest tested PTs' achievement of van Hiele levels 1, 2, 3, and 4, with item numbers 1-5, 6-10, 11-15, and 16-20 representing one van Hiele level each. The identification, labeling, and comparison of geometric shapes like triangles, squares, and rectangles were the topics of the first five questions. The next five questions asked students to identify and label the properties of geometric figures; the third set asked them to logically order the previously identified properties of the figures and start to see the connections between them; and the final set asked them to comprehend the importance of deduction and the function of postulates, axioms, theorems, and proof (Pegg, 1995). Appendix A contains sample items. Following the exam, the researchers collected the completed answer sheets and question papers that had been distributed by the researchers. Since a correct response was worth one mark, a maximum score of twenty could be achieved. SPSS version 26 was used to record the scores and compute the percentage scores. The VHGT was graded once more using a different technique that was based on Usiskin's (1982) "3 of 5 correct" success criterion. According to this criterion, a participant was deemed to have mastered a level if they correctly answered at least three out of five items in any one of the four VHGT subtests. The following grading scheme applied to the participants' scores: 1 point was awarded for meeting the criteria on items 1–5 (level 1); 2 points were awarded for meeting the criteria on items 6–10 (level 2); 4 points were awarded for meeting the criteria on items 11–15 (level 3); and 8 points were awarded for meeting the criteria on items 16–20 (level 4). This could result in a maximum score of 1+2+4+8 = 15 points for any participant. The van Hiele levels at which the requirements were satisfied based solely on the weighted sum were ascertained with the aid of this weighted sum. A score of 7, for instance, would mean that the individual satisfied the requirement at levels 1, 2, and 3 (1+2+4 = 7). Based on their responses, the participants were categorized into different van Hiele levels with the use of this grading system. Should a participant receive a weighted sum of 0, it means they have not met any level because they did not receive at least a 3 on any of the VHGT's subtests. The VHGT scores of the participants were used as a proxy for the PTs' geometrical thinking level.

3.3.2. Structured Interview

Six PTs from the sample participated in structured interviews during which they were given seven open-ended tasks related to geometric shapes. These tasks were developed by Burger and Shaughnessy (1986) and were intended to reflect the descriptions of the van Hiele levels. Genz (2006) used these tasks, which the researchers adopted for the study by changing the activity's figure numbering. The tasks are displayed in Appendix B. The tasks involved drawing triangles and quadrilaterals, defining and identifying shapes, sorting shapes, and participating in informal and formal reasoning regarding geometric shapes. It was anticipated that these assignments would reveal the van Hiele levels 1 through 3 characterizations (Burger & Shaughnessy, 1986). To avoid interfering with regular college activities, the individual interviews took place in the participants' classrooms after school hours and lasted roughly 25 to 30 minutes each. Every interview was done by one of the authors.

3.3.3. Validity and Reliability Analysis

The quantitative instrument used in this section of the study was one that had previously been used for validity and reliability purposes in a number of master's and doctoral works (Baffoe & Mereku, 2010; Burger & Shaughnessy, 1986; Knight, 2006; Usiskin, 1982) since it was developed. The split-half method was used to test the validity. The split-half approach necessitates creating a single test with several items. Then, using the even-odd item criterion, these items are divided or split into two parallel halves. Using the Spearman-Brown formula, which is used in reliability testing, the participant scores from these halves were correlated. The reliability coefficient had a value of.76. This number shows that the instrument has a high level of reliability, demonstrating high internal consistency across the research variables, indicating that the variables met Tucker's (1955) recommendation for internal consistency.

3.3.4. Trustworthiness

In ensuring the trustworthiness of our study, we adopted several methodological precautions during the interview process, as recommended by established qualitative research scholars (Golafshani, 2003; Kaya & Aydin, 2016; Patton, 2002). Patton (2002) states that the goal of an interview is to gain insight into an individual's inner world and to understand his or her point of view. These precautions are outlined below: The interview followed a structured design to maintain consistency and reliability. The structured format allowed for standardized data collection across participants, facilitating meaningful comparisons and analysis. Participants were granted autonomy in responding to interview questions. This approach aimed to capture the richness of individual perspectives and inner worlds, aligning with the goal of qualitative research outlined by Patton (2002). The freedom for participants to express themselves in their own words contributed to the authenticity and depth of the data. Participants were encouraged to carefully review their answers during the interview to ensure clarity and accuracy. This practice aimed to reduce potential misunderstandings and enhance the accuracy of the qualitative data collected. The entire interview process was recorded to prevent distortions and ensure fidelity to participants' responses. This recording served as a crucial tool for later analysis, allowing for a comprehensive and accurate interpretation of the qualitative data.

3.4. Data Analysis

For scores and percentage distributions, frequency analysis was employed. Following the application of descriptive statistics and normality analyses based on kurtosis and skewness values, it was concluded that the distribution's values fall within the range of ±1.4 and that it follows a normal distribution. The data are accepted to have a normal distribution since they fall between the ranges specified by Tabachnick and Fidell (2013) and George and Mallery (2010) (-1 to +1; -1.5 to +1.5; -2.0 to +2.0). Using the criteria established by Usiskin (1982), the data were first analyzed in terms of the percentage mean and subsequently in terms of the percentage of participants in each level of the van Hiele levels. Furthermore, the qualitative data analysis process employed in our study was conducted following established procedures in thematic analysis (Braun & Clarke, 2006). This approach allows for the identification, analysis, and reporting of patterns (themes) within the data, contributing to a comprehensive understanding of the participants' responses.

Upon completion of the interviews, the qualitative data, including PTs' drawings, field notes, and audio-taped interviews, were transcribed verbatim. Subsequently, initial codes were generated to capture meaningful segments of the data related to the PTs' perceptions and responses.

The coded data were then subjected to thematic analysis, wherein emerging themes and patterns were identified. Themes were developed through an iterative process, involving multiple rounds of data review and refinement. This allowed for a nuanced interpretation of the PTs' qualitative responses.

To enhance the trustworthiness and reliability of the analysis, multiple researchers independently coded and analyzed the data. Regular meetings and discussions were held to compare interpretations and resolve any discrepancies, ensuring the validity and consistency of the identified themes.

Given the scope limitations of the study, excerpts from two PTs (one female and one male) were selected to exemplify the identified themes. Pseudonyms were assigned to maintain participant anonymity. These excerpts were chosen strategically to provide a representative illustration of the prevalent patterns observed in the larger dataset.

To further ensure the accuracy of the interpretation, member checking was employed. Selected participants were provided with an opportunity to review and validate the interpretation of their responses, contributing to the confirmability and rigor of the findings.

4. Results

4.1. Quantitative Results

The purpose of this study was to use van Hiele's levels of Geometric Thinking to assess the readiness and classify the geometric thinking levels of the PTs in Peki College of Education. The researchers organized the results of the study by first analyzing the general performance of the participants in the van Hiele's levels of Geometric Thinking using descriptive statistics and presented it in a histogram.

4.1.1. Level of Van Hiele geometrical thinking of mathematics PTs

The overall scores of participants in the VHGT items is presented in Table 2.

Table 2
Shows the general performance of participants in the 20 test items

Scores	*Number of Participants (n)*	*Percentage* (%)	*Participants Cumulative*	*Cumulative Percentage* (%)
2	4	1.3	4	1.3
3	7	2.3	11	3.6
4	15	5.0	26	8.6
5	17	5.6	43	14.2
6	34	11.3	77	25.5
7	31	10.3	108	35.8
8	38	12.6	146	48.4
9	49	16.2	195	64.6
10	38	12.6	233	77.2
11	21	7.0	254	84.2
12	17	5.6	271	89.8
13	13	4.3	284	94.1
14	13	4.3	297	98.4
15	5	1.6	302	100.0
Total	302	100		

From Table 2, it is clear that 195 out of 302 participants obtained less than half of the total score. This represents 64.6% of the total number of PTs who took part in the VHGT. Also, 38 of the participants representing 12.6% obtained exactly half the total score. Additionally, 69 of the participants representing 22.8% of the total number performed above the half mark of 10. The majority of PTs scored between 6 and 9, as evidenced by the higher counts in these score ranges. The scores of 8, 9 and 10 have the highest number of participants. About 35.8% of participants scored 7 or below. Approximately 77.2% of participants scored 10 or below. The cumulative percentage reaches 100% at the score of 15, indicating that all participants are accounted for by the end of the table. The distribution appears to be positively skewed, as there are more participants in the higher score ranges. A close look at the PTs' performance was weak.

Table 3
The van Hiele level of geometric thinking level of participants

Van Hiele Level	*Participants Success Criterion*	*Percentage of Success Criterion* (%)
Visualization: Level 1	177	58.6
Analysis : Level 2	96	31.8
Abstraction: Level 3	21	7.0
Deduction: Level 4	8	2.6
Total	302	100

Table 3 showed that the majority of participants were at Visualization Level 1 (58.6%), indicating that a significant proportion of pre-service teachers in the sample have a basic level of geometric thinking focused on visualizing geometric shapes and structures. A notable number of participants were also at Analysis Level 2 (31.8%), suggesting that a substantial portion of pre-service teachers is capable of analyzing geometric properties and relationships. Smaller percentages of participants are at Abstraction Level 3 (7.0%) and Deduction Level 4 (2.6%), indicating a progressively smaller number of pre-service teachers with higher-order thinking skills in abstraction and deduction. The relatively higher percentage at Visualization Level 1 suggests a potential focus area for instructional interventions aimed at developing higher-order thinking skills.

4.1.2. Readiness of Mathematics PTs to take Euclidean Geometry Course

The readiness concept was conceptually and operationally defined in order to facilitate a better and clearer comprehension of the study. The term "readiness" describes the degree to which people, systems, or organizations are prepared to respond to a crisis and carry out a set of predetermined activities. A person's level of readiness is determined by how well they have planned, how well-trained they are, and how much help they have available (Livet et al., 2022). In this study, "readiness" referred to attaining the Level 4 of van Hiele Geometrical level of thinking. In the light of this, the determination of the mathematics STs level of readiness was done employing researchers – made VHGT test total scores mean intervals: very high (16 – 20), high (11 – 15), moderate (6 – 10), and low (0 – 5).

Table 4
Descriptive Statistics on the Total sores of Participants

Participants	*Minimum*	*Maximum*	*Mean*	*Standard Deviation*	*Readiness Level*
302	2	15	8.5695	2.92772	moderate

The Table 4 provides a snapshot of the distribution of total scores among 302 participants, highlighting the range (from 2 to 15), the average (8.5695), and the variability (standard deviation of 2.92772). The lowest total score observed among the participants is 2. Additionally, the highest total score observed among the participants is 15. The readiness level is also mentioned, indicating that, on average, participants have a moderate readiness level.

4.2. Qualitative Results

The data for the PTs' interviews consisted of the PTs' drawings, the interviewer's field notes and the audio taped interviews. The highlights from only two PTs of the sample (one female and one male), employing pseudo names (PT1, PT2, PT3,PT4, PT5 and PT6) to conform to anonymity, are included in order to comply with limitation of length of the study.

To determine the readiness levels of PT1 and PT5, the grading scheme outlined in the methodology were followed. PT1 received 3 points (1+2= 3) out of 15, indicating mastery at van Hiele levels 1 and 2. Similarly, PT5 accumulated 7 points (1+2+4 = 7) out of 15, placing her at van Hiele levels 1, 2 and 3. This provided a clearer link between the participants' performance and their geometric thinking readiness. In addition, excerpts from the interviews have been included, offering insights into PT1 and PT5's approaches to the VHGT. The interviewee responses suggested that many PTs were only attending to the visual characteristics of the shapes. These excerpts provide context to their VHGT scores and shed light on their thought processes during the test. Excerpts such as "they look like triangles" and "they look like squares" were common in conversations with PTs who were at levels 0 and 1. For example, on the activity to identify and name triangles (Appendix B), the following transcript represents data from PT 1 who was a male based on what he marked on the figures:

> Researcher: Why did you put a "T" on No. 5?
> PT1: Because... it is a 'quadrilateral triangle'
> Researcher: Why do you say so?
> PT1: Because "both sides" are equal.
> Researcher: Why didn't you put a "T" for No. 3 and No.7?
> PT1: They do not "look like" triangles.

Figure 2
Sample Transcript of a male student teacher

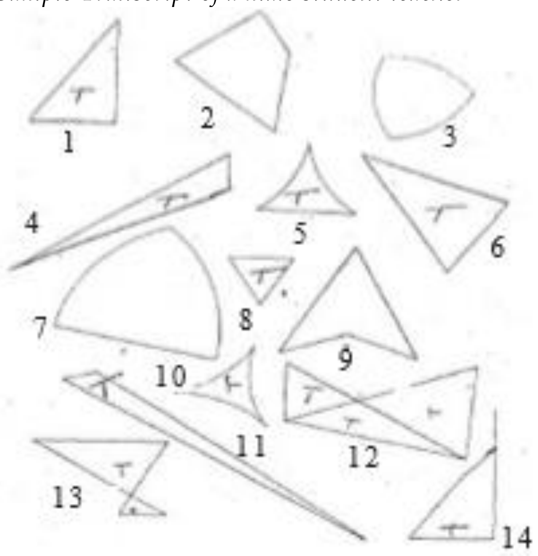
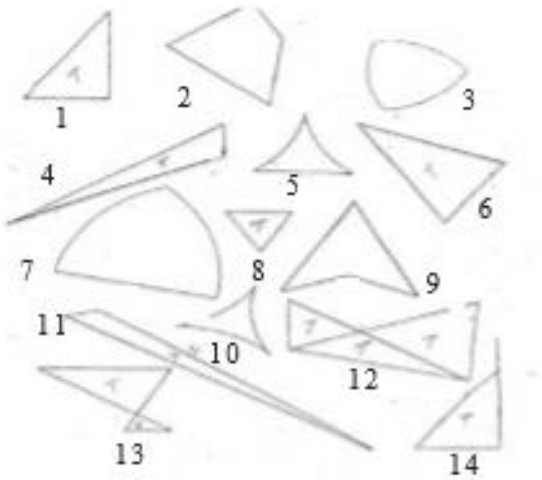


Figure 2 shows that the student did not use the properties when he focused on identifying them (for example, No.5) and could not identify certain triangles. To elicit the properties that the Pre-service teachers perceived as necessary for a figure to be a triangle, the following transcript is evidence:

Researcher: If you want a primary learner to look for a triangle from this paper, what will you tell him or her to look for?
PT1,: Oooo, I will tell him or her to look for 'a figure with 3 sides'.
Researcher: What if the learner picks No.3 and No.7 also?
PT1: Shhhh the learner must look for a 'triangle with 3 sides' like this one here (pointing to the figures he marked).
The following transcript is from the interview with PT,5 who was a female on the same activity:

Figure 2
Sample Transcript of a female student teacher



Researcher: Hi dear, why did you put a "T" on No.4, and No.6?
PT5: ohh sir... Because they are triangles since they have three sides.
Researcher: then why didn't you put a "T" on No.5 and No. 10?
PT5: they are not triangles.
Researcher: Why is No. 7 not a triangle?
PT5: yeah to 'it looks like a circular shape... but not pointy'.
Researcher: How is No.3 different from No.1?
PT5: No.1 'has straight edges and a 90° angle whiles No.3 has a curved edges.

5. Discussion

This present study aimed to assess the readiness of mathematics PTs to take Euclidean geometry course in the new four year B.Ed. mathematics curriculum in Ghana. The major result of this study is summarized as follows:

a) The results of the study indicates a distribution of pre-service teachers across different van Hiele levels, with a predominant focus on Visualization and Analysis. The smaller percentages at higher levels suggest potential areas for targeted educational interventions to enhance abstraction and deduction skills among pre-service teachers.

b) Mathematics pre – service teachers at Peki College of Education exhibited a moderate level of readiness for the upcoming Euclidean geometry course, as indicated by a mean score of 8.5695 and a standard deviation of 2.92772.

c) It was evident from the present study that the majority of the PTs was also not reaching the level set by the four year B.Ed curriculum, which expected the learners to be operating at level 4.

This study's result sheds light on the readiness of PTs to take Euclidean geometry as well as geometry in the new four year mathematics curriculum in Ghana. The study has also added originality to the body of research on educational approaches of teaching Euclidean geometry or geometry in the Ghanaian mathematics education. While earlier studies looked at how van Hiele geometric theory affects pre-service teachers mathematics performance. The present study sought to assess the pre – service teachers' readiness to take Euclidean geometry. The study was distinctive since it concentrated on the college of education environment in Ghana.

The results provided a detailed breakdown of the van Hiele levels, indicating that a significant portion of participants (58.6%) were at Visualization Level 1. This dominance at the foundational level might indicate a need for targeted instructional strategies to enhance higher-order thinking skills, particularly in abstraction and deduction. Numerous researches that looked into the van Hiele theory on students' thinking level of students is consistent with our results (Siyepu, 2005; Usiskin, 1982). In accordance with the results of our study, Asemani (2017) conducted a research in Ghana and found out that majority of the students reached van Hiele level 1 which is the visualization stage where geometric figures are identified based on their appearance. Additionally, Armah et al. (2017) earlier found similar trends in the geometrical thinking abilities of pre-service teachers. More recent studies by Meng and Idris (2012), Abu and Abidin (2013), Meng and Sam (2013) and Ball (2014) also reported similar observations. It was evident from the present study that the majority of the PTs was also not reaching the level set by the curriculum, which expected the PTs to be operating at level 4 and level 5. Ghanaian researchers reports (for example, Armah et al., 2017; Asemani, 2017; Halat & Sahin, 2008; Pandisco & Knight, 2010) indicated that high school learners in general and more especially, senior high school learners are functioning below the levels that are expected of them, that is, they are at concrete and visual levels rather than at abstract level in geometry.

The interview transcript of PT1 indicates that he had not reached visual, analysis, and informal deduction levels of thinking. For him, the properties that he perceived as necessary for a figure to be a triangle were not clear; anything that "looks like a triangle" was a triangle, and No. 5 was a "quadrilateral triangle" because "both sides were equal." The qualitative insights from the interviews highlighted the importance of clarifying geometric terminology and reinforcing formal definitions to enhance readiness levels. In PT5, even though she was able to recognize some triangles, she did not apply the properties when concentrating on doing so, suggesting that she was still not thinking at the levels of analysis and informal deduction. Given that the PTs only focused on the visual prototypes in order to characterize shapes, the two sample responses suggest that the PTs function at either level 1 or level 2. It was common practice to compare shapes using imprecise properties. This conclusion is in line with past research that documented the traits of level 2 thinking (Burger & Shaughnessy, 1986; Pegg & Davey, 1998; van de Walle, 2001; van Hiele 1986). An analysis of interviewees' responses revealed that a number of PTs seemed to have difficulty ordering the properties of simple geometric shapes. The data supported the claims made by Burger (1985) and Mayberry (1983) that many high school students relied on imprecise characteristics to distinguish between shapes, such as "slanted squares" and "pointy triangles," and that they were unable to recognize the characteristics of shapes. Numerous middle school students have significant misconceptions regarding a few important geometric concepts, according to prior research (Burger, 1985; Burger & Shaughnessy, 1986; Feza & Webb, 2005; Fuys et al., 1988; King, 2003; Mayberry, 1983; Renne, 2004; Usiskin 1982) among others. The most insightful thinkers in the interviews were level 0 and level 1. There is an urgent need to address geometry instruction at the college of education because most mathematics PTs at levels 200, 300, and 400 fall below van Hiele level 4, which is the prerequisite for taking Euclidean geometry. Teachers in basic schools need to be at least level 3 in order to perform well (Armah et al., 2017; Crowley, 1987; Usiskin, 1982). Further analysis reveals that 8 of the 302 PTs, or 2.6% of the total, fulfill the prerequisites to enrol in a Euclidean geometry course at a college of education. This suggests that all PTs below the van Hiele level 3 will find learning Euclidean geometry to be very challenging. These findings support the findings of Baffoe and Mereku (2010) and Armah et al. (2017), who noted that learners find van Hiele level 3 to be the most difficult because PTs don't understand that squares are actually rectangles. PTs only had a partial understanding of geometry reasoning at level 4 of the VGHT. According to this study, levels 1 and 2 receive the majority of the attention in geometry instruction, with levels 3 through 5 receiving little to no attention at all.

6. Conclusion and Recommendation

In conclusion, this study sheds light on the geometrical thinking levels of mathematics pre-service teachers and the readiness of mathematics student teachers at Peki College of Education. The quantitative results, unveiled a prevalent basic geometrical thinking among PTs, with a significant majority at Visualization Level 1 and Analysis Level 2. These results are further nuanced and enriched by qualitative insights from sequential explanatory interviews, which revealed a reliance on visual properties and informal language in identifying geometric figures. On the other hand, the study indicates a moderate level of readiness among PTs for the upcoming Euclidean geometry course, as indicated by a mean score of 8.5695 and a standard deviation of 2.92772. This is to suggest that all PTs who are below the van Hiele level 3 will have extreme difficulty in learning Euclidean geometry. The situation is not being compatible with the expectation of the four year B.Ed mathematics curriculum where Euclidean geometry is required to be taught and learnt up to van Hiele's level 4 (Deduction). Mathematics at the Higher institutions is more abstract and Conceptual, and heavily founded on the basis of deductions. For this reason, any student who enters these institutions of higher learning is expected to think at van Hiele's level 4.

Interviews with participants underscored the importance of clarifying geometric terminology to enhance readiness levels. The integration of quantitative and qualitative data offers a comprehensive understanding of the participants' perspectives, allowing for a more nuanced interpretation of the study's results. The essence of this research lies in its contribution to identifying key areas for instructional interventions, emphasizing the need to enhance formal understanding of geometric figures among PTs and reinforcing geometric terminology for improved readiness among Pts. The results have implications for curriculum design and teacher education programs, advocating for strategies that address the specific needs and challenges observed in geometrical thinking and readiness levels. As the study acknowledges its limitations and proposes future research directions, it lays the groundwork for ongoing exploration and improvement in geometry education within teacher preparation programs.

Based on the results of this study, it is recommended that:

- Colleges of Education geometric syllabus should be revived in terms of content and scope in line with current standards.
- Mathematics tutors in Colleges of Education should adopt practical approach to teaching and learning of geometric as a course.
- The supervisory body of Colleges of Education, The National Council for Tertiary Education [NCTE] and the Institute of Education, University of Ghana [UG] should organize seminars and workshops for Mathematics tutors in colleges of Education on van Hiele phase based instruction so that they will be able to incorporate it in their instruction to maximize pre –service mathematics achievement.
- Curriculum developers and writers of Mathematics textbooks should also adopt the van Hiele phase based instruction.

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Appendix A. Sample questions from van Hiele level 1 subtest
Que. 2: Which of these are triangles?

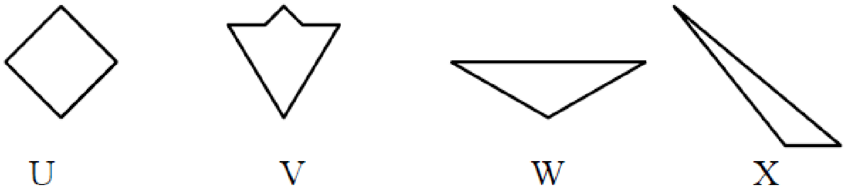

Fig. 1. Sample item from van Hiele level 1 subset.
A. None of these are triangles.
B. V only
C. W only
D. W and X only
E. V and W only

Sample questions from van Hiele level 2 subtest:
Que. 7: In the rectangle GHJK, $\overline{GJ}$ and $\overline{HK}$ are the diagonals.

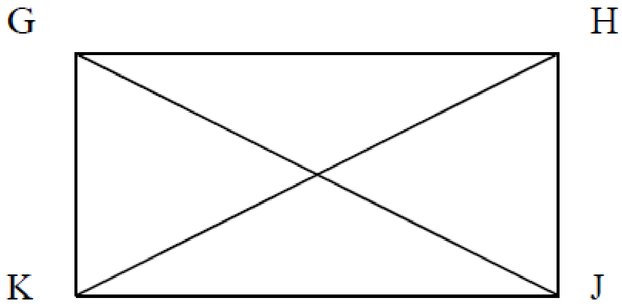

Fig. 2. Sample item from van Hiele level 2 subset.
Which of (A)-(D) is not true in every rectangle?
A. There are four right angles.
B. There are four sides.
C. The diagonals have the same length.
D. The opposite sides have the same length.
E. All of (A)-(D) are true in every rectangle.

Sample questions from van Hiele level 3 subtest:
Que. 10: What do all rectangles have that some parallelograms do not have?
A. Opposite sides equal
B. Diagonals equal
C. Opposite sides parallel
D. Opposite angles equal
E. None of (A)-(D)

Sample questions from van Hiele level 4 subtest:
Question 17: To trisect an angle means to divide it into three parts of equal measure. In 1847, P.L. Wantzel proved that, in general, it is impossible to trisect angles using only a compass and an unmarked ruler. From his proof, what can you conclude?
A. In general, it is impossible to bisect angles using only a compass and an unmarked ruler.
B. In general, it is impossible to trisect angles using only a compass and a marked ruler.
C. In general, it is impossible to trisect angles using any drawing instruments.
D. It is still possible that in the future someone may find a general way to trisect angles using only a compass and an unmarked ruler.
E. No one will ever be able to find a general method for trisecting angles using only a compass and an unmarked ruler.

Appendix B. Pre-Service Teachers Readiness For Euclidean Geometry
Adopted and adapted from Genz (2006: 57 – 58)
Identifying and Defining Triangles (Genz, 2006)

Part A

Aim: to determine whether the student can identify certain triangles.
Instruction: Put a T on each triangle on this sheet

Part B

Aim: to determine the properties that the student focuses on when identifying triangles.
Instruction:

1. Why did you put a T on ……………………………?
2. Are there any triangles in number 12? If so, how many do you see?
3. Are there any triangles in number 13? If so, how many do you see?
4. Pick out at least 4 (if possible) not marked as triangles.
5. From, Q4, why didn't you put T on ………………? (for each one)